ستایش عترت

یازدهم تجربی (پنجشنبه) تکلیف شماره ۲

۱) دامنه؟

الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}\geq 0$

$\frac{(x-1)^2-x^2}{x(x-1)}=\frac{\cancel{x^2}+1-2x-\cancel{x^2}}{x^2-x}\geq 0$

$\frac{-2x+1}{x(x-1)}\geq 0$ $\quad -2x=-1,\ x=\frac{1}{2}$ $\quad 0,\ 1$

$D_f=(-\infty,0)\cup[\frac{1}{2},1)$

$0 \quad \frac{1}{2} \quad 1$

$+ \ | \ - \ 0 \ + \ | \ -$

ب) $f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$ $\quad x\neq -1,\ x\neq 0,\ x\neq 1,\ x\neq -2$

$\frac{3}{x-1}+\frac{1}{x+2}\neq 0 \to x=-\frac{5}{4}$

$\frac{3(x+2)+x-1}{(x-1)(x+2)}=\frac{\overbrace{3x+6+x-1}^{4x+5}}{(x-1)(x+2)}$ $\quad 1,\ -2$

$R-\{-2,-\frac{5}{4},-1,0,1\}$

۲) دامنه؟

الف) $f(x)=\sqrt{[(\frac{1}{3})^x-9](32-4^x)}$

$x=-2 \leftarrow (\frac{1}{3})^x=3^2\to(\frac{1}{3})^{-2}$ $\qquad 32=2^5=4^{\frac{5}{2}}$, $x=\frac{5}{2}$

$-2 \quad \frac{5}{2}$

$+ \ 0 \ - \ 0 \ +$

$D_f=(-\infty,-2]\cup[\frac{5}{2},+\infty)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$ $\quad x-1\geq 0\to x\geq 1$

$\sqrt{y+1}=3-\sqrt[4]{x-1}$

$3-\sqrt[4]{x-1}\geq 0$

$\sqrt[4]{x-1}\leq 3 \xrightarrow{\text{توان ۴}} x-1\leq 81$

$x\leq 82$

$D_f=[1,82]$

۳) دامنه؟

$f(x)=\frac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1}$ $\quad >0\to(x-2)(x+1)>0$ $\quad 2,\ -1$

$-1 \quad 2$

$+ \ 0 \ - \ 0 \ +$

$-1 \quad +1 \quad +2$

$x^2-1\geq 0$

$D_f=(-\infty,-1)\cup(2,+\infty)$ $\quad x^2\geq 1\to x\geq 1,\ x\leq -1$

۴) دامنه؟

$(x+2) \qquad (x-b)$

$\sqrt{3+ax-x^2}\geq 0\to D_f=[-2,b]$ $\qquad a+b=?$ 1]

$x^2-ax-3\leq 0$

$(x+2)(x-b)\leq 0$

$-2 \qquad +b$

$-b\times -2=-3$

$b=\frac{3}{2}$

$a=\frac{3}{2}-2=\frac{3-4}{2}=-\frac{1}{2}$

$a+b=-\frac{1}{2}+\frac{3}{2}=+\frac{2}{2}=+1$]

۵) دامنه؟

$$f(x)=\begin{cases} 3x-2 & x\geq 1 \\ 2x+3 & x<1 \end{cases} \qquad g(x)=\sqrt{f(x)-x}$$

$x\geq 1 \to 3x-2-x=2x-2\geq 0 \to 2x\geq 2$, $x\geq 1$ ✓

$x<1 \to 2x+3-x=x+3\geq 0 \to x\geq -3$

$[-3,1)$ ✓

$D_g=[-3,+\infty)$

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۶) مقدار a؟

$$f(x)=\begin{cases} (a+1)(x+2) & x>1 \\ 3a+2x & x\leq 1 \end{cases} \qquad 2f(5)=f(-2)+a$$

$\xrightarrow{5>1}\ 2f(5)=\underbrace{2(a+1)}_{14}\underbrace{(\overset{5}{x}+2)}_{7}=14(a+1)=14a+14$

$\xrightarrow{-2\leq 1}\ f(-2)=3a-4$

$14a+14=3a-4+a$

$14a+14=4a-4$

$10a=-18 \longrightarrow a=-\frac{18}{10}=\frac{-9}{5}$

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۷)

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2 \qquad f(\overset{x_1}{2-\sqrt{3}})+f(\overset{x_2}{2+\sqrt{3}})$$

$(2+\sqrt{3})\times(2-\sqrt{3})=4-3=\boxed{1} \to$ معکوس هم هستند

$\sqrt{x_1}+\frac{1}{\sqrt{x_1}}+\sqrt{x_2}+\frac{1}{\sqrt{x_2}}+4$; $\frac{1}{\sqrt{x_1}}\to\sqrt{x_2}$, $\frac{1}{\sqrt{x_2}}\to\sqrt{x_1}$

$=2\sqrt{x_1}+2\sqrt{x_2}+4=2\underbrace{(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}})}_{A}+4$

$A^2=\overbrace{(2-\sqrt{3})+(2+\sqrt{3})}^{4}+2\sqrt{\underbrace{(2+\sqrt{3})(2-\sqrt{3})}_{1}}=4+2=6$

$A=\sqrt{6}$

$\Rightarrow \boxed{2\sqrt{6}+4}$

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۸)

$2f(x)-3f(-x)=4x^2-x$

$f(x)=?$

$\xrightarrow{x}(2f(x)-3f(-x)=4x^2-x)\times 2$

$\xrightarrow{-x}(2f(-x)-3f(x)=4x^2+x)\times 3$

$4f(x)-6\cancel{f(-x)}=8x^2-2x$

$6\cancel{f(-x)}-9f(x)=12x^2+3x$

$-5f(x)=20x^2+x \to f(x)=\frac{20x^2+x}{-5}$

$f(x)=-4x^2-\frac{x}{5}$

$f(0) = ?$ $\qquad (x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$

(۹)

$x = 0 \longrightarrow 2f(0) = 3m - 1 \longrightarrow f(0) = \dfrac{3m-1}{2}$

$x = -2 \longrightarrow 6f(0) = 16 + 2m + 3m - 1$

$6f(0) = 15 + 5m \longrightarrow f(0) = \dfrac{5m+15}{6}$

$\dfrac{3m-1}{\cancel{2}_{1}} = \dfrac{5m+15}{\cancel{6}_{3}}$

$\cancel{9m} - 3 = \cancel{5m} + 15$

$4m = 18$

$2m = 9 \longrightarrow m = \dfrac{9}{2}$

$f(0) = \dfrac{3 \times \frac{9}{2} - 1}{2} = \boxed{\dfrac{25}{4}}$

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(۱۰)

f خطی $\longrightarrow ax+b$

$f(x) \longrightarrow f\left(\dfrac{1}{x}\right) = \dfrac{3x^2 - 12x + 3}{x}$

$f(-1) = ?$

$\longrightarrow x = -1 \Rightarrow 2f(-1) = \dfrac{3 + 12 + 3}{-1}$

$2f(-1) = -18$

$f(-1) = \boxed{-9}$