ستایش عتره — یازدهم دختر (پنجشنبه) — تکلیف شماره ۴

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۱) دانشم

الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}\geq 0$

$$\frac{(x-1)^2-x^2}{x(x-1)}=\frac{\cancel{x^2}+1-2x-\cancel{x^2}}{x^2-x}\geq 0$$

$$\frac{-2x+1}{x(x-1)}\geq 0 \qquad -2x=-1,\quad x=\frac12 \qquad x=0,\ x=1$$

$$D_f=(-\infty,0)\cup[\tfrac12,1)$$

0	$\frac12$	1
+	−	+	−

ب) $f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$ $\quad x\neq -1,\ x\neq 0,\ x\neq 1,\ x\neq -2$

$$\frac{3}{x-1}+\frac{1}{x+2}\neq 0 \longrightarrow x=-\frac54$$

$$\frac{3(x+2)+x-1}{(x-1)(x+2)}=\frac{\overbrace{3x+6+x-1}^{4x+5}}{(x-1)(x+2)}$$

$1,\ -2$

$$R-\{-2,-\tfrac54,-1,0,1\}$$

۲) دانشم

الف) $f(x)=\sqrt{\left[(\tfrac13)^x-9\right](32-4^x)}$

$$x=-2 \leftarrow (\tfrac13)^x=3^2 \to (\tfrac13)^{-2} \qquad 32=2^5=4^{\frac52},\quad x=\frac52$$

−2	$\frac52$
+	−	+

$$D_f=(-\infty,-2]\cup[\tfrac52,+\infty)$$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$ $\qquad x-1\geq 0\to x\geq 1$

$$\sqrt{y+1}=3-\sqrt[4]{x-1}$$

$$3-\sqrt[4]{x-1}\geq 0$$

$$\sqrt[4]{x-1}\leq 3 \xrightarrow{\text{توان ۴}} x-1\leq 81,\quad x\leq 82$$

$$D_f=[1,82]$$

۳) دانشم

$$f(x)=\frac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1}$$

$$\log_4(x^2-x-2)>0 \to (x-2)(x+1)>0 \qquad 2,\ -1$$

−1	2
+	−	+

$-1 \quad +1 \quad +2$

$$x^2-1\geq 0 \qquad x^2\geq 1 \to x\geq 1,\ x\leq -1$$

$$\boxed{D_f=(-\infty,-1)\cup(2,+\infty)}$$

۴) دانشم

$$\sqrt{3+ax-x^2}\geq 0 \longrightarrow D_f=[-2,b] \qquad a+b=?$$

$(x+2)\qquad(x-b)$

$$x^2-ax-3\leq 0$$

$$(x+2)(x-b)\leq 0 \qquad -2,\ +b$$

$$-b\times -2=-3$$

$$b=\frac32$$

$$a=\frac32-2=\frac{3-4}{2}=-\frac12$$

$$a+b=-\frac12+\frac32=+\frac22=+1$$

$$f(x)=\begin{cases}3x-2 & x\geq 1\\ 2x+3 & x<1\end{cases}\qquad g(x)=\sqrt{f(x)-x}$$

دامنه ؟ ۵

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$x\geq 1 \to 3x-2-x=2x-2\geq 0 \to 2x\geq 2$

$x\geq 1$ ✓

$x<1 \to 2x+3-x=x+3\geq 0 \to x\geq -3$

$D_g=[-3,+\infty)$

$[-3,1)$ ✓

$$f(x)=\begin{cases}(a+1)(x+2) & x>1\\ 3a+2x & x\leq 1\end{cases}\qquad 2f(5)=f(-2)+a$$

مقدار a ؟ ۴

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$\xrightarrow{5>1}\ 2f(5)=\underbrace{2(a+1)(\overset{5}{x}+2)}_{14}=14(a+1)=14a+14$

$\xrightarrow{-2\leq 1}\ f(-2)=3a-4$

$14a+14=3a-4+a$

$14a+14=4a-4$

$10a=-18 \longrightarrow a=-\frac{18}{10}=\frac{-9}{5}$

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2\qquad f(\overset{x_1}{2-\sqrt{3}})+f(\overset{x_2}{2+\sqrt{3}})$$

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$(2+\sqrt{3})\times(2-\sqrt{3})=4-3=1 \to$ معکوس هم هستند

$\sqrt{x_1}+\frac{1}{\sqrt{x_1}}\to\sqrt{x_2} \quad +\sqrt{x_2}+\frac{1}{\sqrt{x_2}}\to\sqrt{x_1}\quad +4$

$=2\sqrt{x_1}+2\sqrt{x_2}+4=2(\underbrace{\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}}_{A})+4$

$A^2=\overbrace{(2-\sqrt{3})+(2+\sqrt{3})}^{4}+2\sqrt{\underbrace{(2+\sqrt{3})(2-\sqrt{3})}_{1}}=4+2=6$

$A=\sqrt{6}$

$\Rightarrow \boxed{2\sqrt{6}+4}$

$2f(x)-3f(-x)=4x^2-x$

$f(x)=?$

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$\xrightarrow{x}(2f(x)-3f(-x)=4x^2-x)\times 2$

$\xrightarrow{-x}(2f(-x)-3f(x)=4x^2+x)\times 3$

$4f(x)-6\cancel{f(-x)}=8x^2-2x$

$6\cancel{f(-x)}-9f(x)=12x^2+3x$

$-5f(x)=20x^2+x \longrightarrow f(x)=\frac{20x^2+x}{-5}$

$f(x)=-4x^2-\frac{x}{5}$

$f(0) = ?$ $\qquad (x+2)f(x) - 3x\,f(x+2) = 4x^2 - mx + 3m - 1$ 9 (۹)

$x = 0 \longrightarrow 2f(0) = 3m - 1 \longrightarrow f(0) = \frac{3m-1}{2}$

$x = -2 \longrightarrow 4f(0) = 16 + 2m + 3m - 1$

$4f(0) = 15 + 5m \longrightarrow f(0) = \frac{5m+15}{4}$

$\frac{3m-1}{\cancel{2}_{1}} = \frac{5m+15}{\cancel{4}_{2}}$

$\cancel{6}m - 3 = \cancel{5}m + 15$

$4m = 18$

$2m = 9 \longrightarrow m = \frac{9}{2}$

$f(0) = \frac{3 \times \frac{9}{2} - 1}{2} = \boxed{\frac{25}{4}}$

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9 (۱۰)

f خطی $\longrightarrow ax + b$

$f(x) \longrightarrow f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x} \longrightarrow x = -1 \Rightarrow 2f(-1) = \frac{3 + 12 + 3}{-1}$

$f(-1) = ?$

$2f(-1) = -18$

$f(-1) = \boxed{-9}$
