

الف) $f(n) = \sqrt{\frac{n-1}{n} - \frac{n}{n-1}} = \sqrt{\frac{(n-1)^2 - n^2}{n(n-1)}} = \sqrt{\frac{(n-1-n)(n-1+n)}{n(n-1)}} = \sqrt{\frac{-(2n-1)}{n(n-1)}}$

$\frac{-(2n-1)}{n(n-1)} \geq 0$ $\Rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(n) = \frac{1}{n+1} - \frac{2}{n}$ $\Rightarrow D_f = \mathbb{R} - \{-2, \pm 1, 0, -\frac{1}{2}\}$

ج) $f(n) = \sqrt{((\frac{1}{n})^2 - 9)(32 - 4n)} = \sqrt{(32 - 4n)(\frac{1}{n^2} - 9)} \Rightarrow (32 - 4n)(\frac{1}{n^2} - 9) \geq 0$

$\Rightarrow D_f = (-\infty, -2] \cup [\frac{1}{2}, +\infty)$

د) $\sqrt{x-1} + \sqrt{y+1} = 3$ $\Rightarrow D_f = [1, 12]$

$x-1 \geq 0 \Rightarrow x \geq 1$ $\sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow \sqrt{y+1} \geq 0 \Rightarrow 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$

$f(n) = \frac{\log_2(x^2 - n - 2)}{\sqrt{x^2 - 1} + 1}$

$x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \geq 1$

$x^2 - n - 2 \geq 0 \Rightarrow (n-2)(n+1) \geq 0$

$\sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{x^2 - 1} \neq -1$

$D_f = (-\infty, -1) \cup (2, +\infty)$

$f(n) = \sqrt{3 + an - n^2}$

$D_f = [-2, b]$

$3 - 2a - f \geq 0 \Rightarrow a = \frac{1}{2}$

$f(n) = \sqrt{-n^2 - \frac{1}{2}n + 3}$ $\Rightarrow \frac{3}{-1} = (-2) \times b \Rightarrow b = \frac{3}{2}$

$b + a = \frac{3}{2} + (-\frac{1}{2}) = 1$

$f(n) = \begin{cases} 2n-2 & ; n \geq 1 \\ 2n+2 & ; n < 1 \end{cases}$

$g(n) = \sqrt{f(n) - n}$

$f(n) - n \geq 0 \Rightarrow f(n) \geq n$

$2n-2 \geq n \Rightarrow n \geq 2$

$2n+2 \geq n \Rightarrow n \geq -2$

$D_g = [-2, +\infty)$

$$f(n) = \begin{cases} (a+1)(n+r) & ; n > 1 \\ \sqrt{a} + \sqrt{n} & ; n \leq 1 \end{cases}$$

$$\left. \begin{aligned} \sqrt{f(a)} &= \sqrt{(a+1)(a+r)} = \sqrt{a+1} \sqrt{a+r} \\ f(-r) &= \sqrt{a} - \sqrt{r} \end{aligned} \right\} \rightarrow \sqrt{f(a)} = f(-r) + a \Rightarrow \sqrt{a+1} \sqrt{a+r} = \sqrt{a} - \sqrt{r} + a$$

$$\rightarrow 10a = -1 \rightarrow \boxed{a = -1, 1}$$

$$f(n) = \sqrt{n} + \frac{1}{\sqrt{n}} + r$$

$$\left. \begin{aligned} f(r+\sqrt{r}) &= \sqrt{r+\sqrt{r}} + \frac{1}{\sqrt{r+\sqrt{r}}} + r \\ f(r-\sqrt{r}) &= \sqrt{r-\sqrt{r}} + \frac{1}{\sqrt{r-\sqrt{r}}} + r \end{aligned} \right\} \Rightarrow A = \sqrt{r+\sqrt{r}} + \sqrt{r-\sqrt{r}} + \frac{1}{\sqrt{r+\sqrt{r}}} + \frac{1}{\sqrt{r-\sqrt{r}}} + r$$

$$\Rightarrow A = \sqrt{4} + \sqrt{r} + \sqrt{4} - \sqrt{r} + r = \boxed{A = r\sqrt{4} + r}$$

$$\sqrt{f(n)} - \sqrt{f(-n)} = \sqrt{n^r - n} \xrightarrow{\times r} \sqrt{f(n)} - \sqrt{f(-n)} = An^r - r$$

$$\sqrt{f(-n)} - \sqrt{f(n)} = \sqrt{n^r + n} \xrightarrow{\times r} -\sqrt{f(n)} + \sqrt{f(-n)} = 12n^r + r$$

$$\textcircled{1} + \textcircled{2} \Rightarrow -2\sqrt{f(n)} = 10n^r + n \rightarrow \boxed{f(n) = n^r - \frac{1}{2}n}$$

$$(n+r)f(n) - \sqrt{n}f(n+r) = \sqrt{n^r - mn} + \sqrt{m} - 1$$

$$\xrightarrow{n=0} \sqrt{f(0)} = \sqrt{m} - 1 \rightarrow f(0) = \frac{m-1}{r}$$

$$\xrightarrow{n=r} \sqrt{f(0)} = 14 + \sqrt{m} + \sqrt{m} - 1 \rightarrow \sqrt{f(0)} = 2m+13 \rightarrow f(0) = \frac{2m+13}{4}$$

$$\Rightarrow \frac{m-1}{r} = \frac{2m+13}{4} \Rightarrow 4m-4 = 2m+13 \rightarrow m = \frac{17}{2} = 8.5$$

$$\boxed{f(0) = \frac{m-1}{r} = \frac{8.5-1}{4} = \frac{7.5}{4}}$$

$$\left. \begin{aligned} f(n) &= an+b \\ f\left(\frac{1}{n}\right) &= \frac{a}{n}+b \end{aligned} \right\} f(n) + f\left(\frac{1}{n}\right) = an+b + \frac{a}{n}+b = \frac{an^2 + 12n + a}{n} = \frac{m^r - 12n + r}{n}$$

$$\Rightarrow a=3, b=-9 \rightarrow f(n) = 3n-9$$

$$\Rightarrow f(-1) = 3(-1)-9 = \boxed{-12}$$

$$f(n) + f\left(\frac{1}{n}\right) = \frac{m^r - 12n + r}{n} \xrightarrow{n=1} \frac{f(-1) + f(-1)}{2f(-1)} = \frac{m+12+r}{-1}$$

$$\Rightarrow \sqrt{f(-1)} = -11 \rightarrow \boxed{f(-1) = -12}$$