

الف) $f(n) = \sqrt{\frac{n-1}{n}} - \frac{n}{n-1} = \sqrt{\frac{(n-1)^2 - n^2}{n(n-1)}} = \sqrt{\frac{(n-1-n)(n-1+n)}{n(n-1)}} = \sqrt{\frac{-(2n-1)}{n(n-1)}}$

$\frac{-(2n-1)}{n(n-1)} \geq 0$

Sign chart: $\frac{0}{+} \quad \frac{1}{-}$

$\Rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(n) = \frac{1}{n+1} - \frac{2}{n}$

Sign chart: $\frac{1}{+} \quad \frac{2}{-}$

$\frac{1}{n+1} - \frac{2}{n} \geq 0 \Rightarrow \frac{n - 2(n+1)}{n(n+1)} \geq 0 \Rightarrow \frac{-n-2}{n(n+1)} \geq 0$

$\Rightarrow D_f = \mathbb{R} - \{-2, \pm 1, 0, -\frac{2}{n}\}$

ج) $f(n) = \sqrt{((\frac{1}{n})^2 - 9)(32 - 4n)} = \sqrt{(32 - 4n)(\frac{1}{n^2} - 9)} \Rightarrow (32 - 4n)(\frac{1}{n^2} - 9) \geq 0$

Sign chart: $\frac{-2}{+} \quad \frac{0}{-} \quad \frac{4}{+}$

$\Rightarrow D_f = (-\infty, -2] \cup [\frac{4}{9}, +\infty)$

د) $\sqrt{x-1} + \sqrt{y+1} = 3$

$\Rightarrow D_f = [1, 8]$

چون $x-1 \geq 0 \Rightarrow x \geq 1$ و $y+1 \geq 0 \Rightarrow y \geq -1$

$\sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$

با این که $x \geq 1$ پس $1 \leq x \leq 10$

$f(n) = \frac{\log_2(n^2 - n - 2)}{\sqrt{n^2 - 1} + 1}$

$n^2 - 1 \geq 0 \Rightarrow n^2 \geq 1 \Rightarrow n \geq 1$

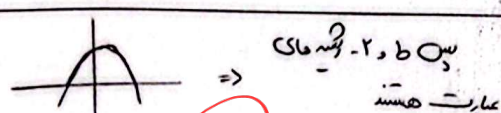
$n^2 - n - 2 \geq 0 \Rightarrow (n-2)(n+1) \geq 0 \Rightarrow n \geq 2$

$\sqrt{n^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{n^2 - 1} \neq -1$

$\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$f(n) = \sqrt{3 + an - n^2}$

$D_f = [-2, b]$



$n = -2 \Rightarrow 3 - 4a - 4 \geq 0 \Rightarrow a \leq \frac{1}{4}$

$f(n) = \sqrt{-n^2 - \frac{1}{4}n + 3} \Rightarrow \frac{1}{4}n + 3 \geq 0 \Rightarrow n \geq -12$

$b + a = \frac{3}{4} + (-\frac{1}{4}) = \frac{1}{2} \Rightarrow b = \frac{1}{2} - a$

$f(n) = \begin{cases} 2n-2 & ; n \geq 1 \\ 2n+2 & ; n < 1 \end{cases}$

$g(n) = \sqrt{f(n) - n}$

$f(n) - n \geq 0 \Rightarrow f(n) \geq n$

Case 1: $2n-2 \geq n \Rightarrow n \geq 2$

Case 2: $2n+2 \geq n \Rightarrow n \geq -2$

$\Rightarrow D_g = [-2, +\infty)$

$$f(n) = \begin{cases} (a+1)(n+r) & ; n > 1 \\ ra + rn & ; n \leq 1 \end{cases}$$

$$\left. \begin{aligned} rf(a) &= r(a+1)(a+r) = r(a+1)r \\ f(-r) &= ra - r \end{aligned} \right\} \rightarrow rf(a) = f(-r) + a \Rightarrow |ra+1r = ra-r+a$$

$$\rightarrow 10a = -1 \rightarrow \boxed{a = -1, 1}$$

$$f(n) = \sqrt{n} + \frac{1}{\sqrt{n}} + r$$

$$\left. \begin{aligned} f(r+\sqrt{r}) &= \sqrt{r+\sqrt{r}} + \frac{1}{\sqrt{r+\sqrt{r}}} + r \\ f(r-\sqrt{r}) &= \sqrt{r-\sqrt{r}} + \frac{1}{\sqrt{r-\sqrt{r}}} + r \end{aligned} \right\} \Rightarrow A = r\sqrt{r+\sqrt{r}} + r\sqrt{r-\sqrt{r}} + r$$

$$\Rightarrow A = \sqrt{4} + \sqrt{r} + \sqrt{4} - \sqrt{r} + r \Rightarrow \boxed{A = r\sqrt{4} + r}$$

$$rf(n) - rf(-n) = rn^r - n \xrightarrow{\times r} rf(n) - 4f(-n) = An^r - rn \quad \textcircled{1}$$

$$rf(-n) - rf(n) = rn^r + n \xrightarrow{\times r} -4f(n) + 4f(-n) = 12n^r + rn \quad \textcircled{2}$$

$$\textcircled{1} + \textcircled{2} \Rightarrow -2f(n) = 10n^r + n \rightarrow \boxed{f(n) = -5n^r - \frac{1}{2}n}$$

$$(n+r)f(n) - rn f(n+r) = rn^r - mn + rn - 1$$

$$\xrightarrow{n=0} rf(0) = rn - 1 \rightarrow f(0) = \frac{rn-1}{r}$$

$$\xrightarrow{n=r} 4f(0) = 14 + rn + rn - 1 \rightarrow 4f(0) = 2m + 13 \rightarrow f(0) = \frac{2m+13}{4}$$

$$\Rightarrow \frac{rn-1}{r} = \frac{2m+13}{4} \Rightarrow 4m - r = 2m + 13 \rightarrow m = \frac{14}{2} = 7$$

$$\boxed{f(0) = \frac{rn \times \frac{9}{r} - 1}{r} = \frac{9a}{r}}$$

$$\left. \begin{aligned} f(n) &= an + b \\ f\left(\frac{1}{n}\right) &= \frac{a}{n} + b \end{aligned} \right\} f(n) + f\left(\frac{1}{n}\right) = an + b + \frac{a}{n} + b = \frac{an^2 + 1bn + a}{n} = \frac{rn^2 - 1rn + r}{n}$$

$$\Rightarrow a = r, b = -9 \rightarrow f(n) = rn - 9$$

$$\Rightarrow f(-1) = r(-1) - 9 = \boxed{-9}$$

$$f(n) + f\left(\frac{1}{n}\right) = \frac{rn^2 - 1rn + r}{n} \xrightarrow{n=1} \frac{f(-1) + f(-1)}{rf(-1)} = \frac{r+1r+r}{-1}$$

$$\Rightarrow rf(-1) = -11 \rightarrow \boxed{f(-1) = -9}$$