۱-

الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}=\sqrt{\frac{x^2-2x+1-x^2}{x^2-x}}=\sqrt{\frac{1-2x}{x(x-1)}}$

$0 \quad \frac{1}{2} \quad 1$

$+ \quad - \quad + \quad -$

$D_f=(-\infty,0)\cup[\frac{1}{2},1)$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}=\dfrac{\frac{x-2x-2}{x(x+1)}}{\frac{3x+6+x-1}{(x-1)(x+2)}}\to 4x+5\neq 0\Rightarrow x\neq-\frac{5}{4}$

$x\neq\{0,-1,1,-2\}$

$D_f=\mathbb{R}-\{0,\pm1,-2,-\frac{5}{4}\}$

۲-

الف) $f(x)=\sqrt{((\frac{1}{3})^x-9)(32-4^x)}\to 2^{2x}=2^5\Rightarrow 2x=5\Rightarrow x=\frac{5}{2}$

$-2=$ ریشه   ریشه $=\frac{5}{2}$

$-2 \quad \frac{5}{2}$

$+ \quad - \quad +$

$\to D_f=(-\infty,-2]\cup[\frac{5}{2},+\infty)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3\Rightarrow\sqrt{y+1}=3-\sqrt[4]{x-1}$

$3-\sqrt[4]{x-1}\geq 0\Rightarrow\sqrt[4]{x-1}\leq 3\Rightarrow x-1\leq 81\Rightarrow x\leq 82$ $x-1\geq 0\Rightarrow x\geq 1$

$D_y=[1,82]$

۳-

$f(x)=\dfrac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1}$ $x^2-x-2>0\Rightarrow(x-2)(x+1)>0$

$-1 \quad 2$

$+ \quad - \quad +$

$(-\infty,-1)\cup(2,+\infty)$

$x^2-1>0\Rightarrow(x-1)(x+1)>0$

$-1 \quad 1$

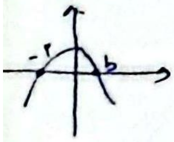
$+ \quad - \quad +$

$(-\infty,-1)\cup(1,+\infty)$

$\to D_f=(-\infty,-1)\cup(2,+\infty)$

۴-

$\sqrt{-x^2+ax+3}$ $D_f=[-2,b]$ $\xrightarrow{x=-2} -4-2a+3=0\Rightarrow 2a=-1\Rightarrow a=-\frac{1}{2}$



$\to\sqrt{-x^2-\frac{1}{2}x+3}$ $-x^2-\frac{1}{2}x+3=0\Rightarrow x^2+\frac{1}{2}x-3=0$

$\Rightarrow 2x^2+x-6=0$ $\frac{-1\pm\sqrt{1+48}}{4}$ ($\sqrt{1+48}=7$), $x=-\frac{8}{4}=-2$, $b=x=\frac{6}{4}=\frac{3}{2}$

$a+b=-\frac{1}{2}+\frac{3}{2}=\frac{2}{2}=1$

۵-

$$f(x)=\begin{cases}3x-2 & ; x\geq 1\\ 2x+3 & ; x<1\end{cases}$$

$$g(x)=\sqrt{f(x)-x} \Rightarrow f(x)-x\geq 0 \Rightarrow \left.\begin{array}{l}3x-2-x\geq 0 \Rightarrow 2x\geq 2 \Rightarrow x\geq 1\\ 2x+3-x\geq 0 \Rightarrow x+3\geq 0 \Rightarrow x\geq -3\end{array}\right\} \xrightarrow{\cap} [-3,+\infty)$$

۶-

$$f(x)=\begin{cases}(a+1)(x+2) & ; x>1\\ 3a+2x & ; x\leq 1\end{cases}$$

$$2f(5)=f(-2)+a \Rightarrow 2\left(\overbrace{(a+1)(7)}^{7a+7}\right)=3a-4+a \Rightarrow 14a+14=4a-4$$

$$10a=-18 \Rightarrow \boxed{a=-1.8}$$

۷-

$$f(2-\sqrt{3})=\sqrt{2-\sqrt{3}}+\frac{1}{\sqrt{2-\sqrt{3}}}+2 \quad \frac{1}{\sqrt{2-\sqrt{3}}}\times\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2+\sqrt{3}}}=\frac{\sqrt{2+\sqrt{3}}}{\sqrt{1}}=\sqrt{2+\sqrt{3}}$$

$$f(2+\sqrt{3})=\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}+2$$

$$\sqrt{2-\sqrt{3}}=\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\times\frac{1}{\sqrt{2+\sqrt{3}}}$$

$$\oplus \quad \sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}+2+\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}+2 \Rightarrow 2(\underbrace{\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}}_{t})+4$$

$$t^2=2-\cancel{\sqrt{3}}+2+\cancel{\sqrt{3}}+2(\sqrt{1})=6 \Rightarrow t=\sqrt{6} \Rightarrow \sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}=\sqrt{6}$$

$$\Rightarrow f(2-\sqrt{3})+f(2+\sqrt{3})=\boxed{2\sqrt{6}+4}$$

۸-

$$\begin{array}{ll}2f(x)-3f(-x)=4x^2-x \rightarrow & 4f(x)-6\cancel{f(-x)}=8x^2-2x\\ 2f(-x)-3f(x)=4x^2+x \rightarrow & 6\cancel{f(-x)}-9f(x)=12x^2+3x\\ \hline & -5f(x)=20x^2+x \Rightarrow f(x)=-\dfrac{20x^2+x}{5}\end{array}$$

۹-

$$(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1$$

$$\begin{array}{ll}\xrightarrow{x=0} 2f(0)-0=3m-1 \xrightarrow{\times 5} & 10f(0)=15m-5\\ \xrightarrow{x=-2} 6f(0)=16+\underbrace{2m+3m}_{5m}-1 \Rightarrow & 18f(0)=45+15m\\ \hline & 8f(0)=50\\ & \Rightarrow f(0)=\dfrac{50}{8}=\dfrac{25}{4}=\boxed{6.25}\end{array}$$

۱۰-

$$f(x)+f\left(\frac{1}{x}\right)=\frac{3x^2-12x+3}{x} \qquad f(x)=ax+b$$

$$ax+b+\frac{a}{x}+b=\frac{3x^2-12x+3}{x} \Rightarrow \frac{ax^2+2bx+a}{x}=\frac{3x^2-12x+3}{x}$$

$$a=3 \qquad 2b=-12 \qquad b=-6$$

$$y=ax+b \Rightarrow f(x)=3x-6 \Rightarrow f(-1)=-3-6=-9$$