گروه یازدهم دختران C — به نام خدا — کار ریاضی

1)

الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}$

$\frac{x-1}{x}-\frac{x}{x-1}\geq 0$

$\frac{(x-1)^2-x^2}{x^2-x}=\frac{x^2+1-2x-x^2}{x^2-x}\to\frac{-2x+1}{x(x-1)}\geq 0$

	0	$\frac12$	1	
$+$		$-$	$+$	$-$

$D_{f_1}=(-\infty,0)\cup[\frac12,1)$

ب) $f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$ $\to x\neq -1,\ x\neq 0$

$\frac{3}{x-1}+\frac{1}{x+2}\neq 0$

$\frac{3(x+2)+x-1}{(x-1)(x+2)}\to\frac{4x+5}{(x-1)(x+2)}$

$D_f=\mathbb{R}-\{-2,-\frac54,-1,0,1\}$

2)

الف) $\sqrt{((\frac13)^x-9)(32-4^x)}$

$(\frac13)^x-9=0\to(\frac13)^x=9\to x=-2$

$32-4^x=0\to 2^{2x}=2^5\to x=\frac52$

	-2	$\frac52$	
$+$		$-$	$+$

$D_f=(-\infty,-2]\cup[\frac52,+\infty)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$\sqrt{y+1}=3-\sqrt[4]{x-1}\Rightarrow 3-\sqrt[4]{x-1}\geq 0$

$\sqrt[4]{x-1}\leq 3\xrightarrow{\text{توان}} x-1\leq 81\to x\leq 82$

$x-1\geq 0\to x\geq 1$

$D_f=[1,82]$

3)

$f(x)=\frac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1}$

$x^2-x-2>0\to(x-2)(x+1)>0$, $x=2$, $x=-1$

	-1	2	
$+$		$-$	$+$

$x^2-1\geq 0$

$x^2\geq 1\to x\geq 1$, $x\leq -1$

$D_f=(-\infty,-1)\cup(2,+\infty)$

4)

$f(x)=\sqrt{3+ax-x^2}$

$-x^2+ax+3\geq 0$

$x^2-ax-3\leq 0$

$D_f=[-2,b]$

$(x+2)(x-b)\to(x+2)(x-\frac32)$

$-2b=-3$

$b=\frac32$

$x^2+\frac12x-3\leq 0$

$a=-\frac12$

$a+b=\frac32-\frac12=\boxed{1}$

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$$f(x) = \begin{cases} 3x-2 & , x \geq 1 \\ 2x+3 & , x<1 \end{cases} \longrightarrow \begin{aligned} &3x-2 \geq x \\ &2x \geq 2 \\ &x \geq 1 \end{aligned}$$

برای اعداد بزرگتر مساوی یک برقرار است

$$g(x) = \sqrt{f(x)-x} \qquad \begin{aligned} &2x+3 \geq x \\ &x+3 \geq 0 \\ &x \geq -3 \end{aligned}$$

برای اعداد بزرگتر از −۳ همه برقرار است

$$D_g = [-3, +\infty)$$

$$f(x) - x \geq 0 \rightarrow f(x) \geq x$$

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$$f(x) = \begin{cases} (a+1)(x-2) & , x>1 \\ 3a+2x & , x \leq 1 \end{cases}$$

$$2f(5) = f(-2) + a$$

$$2(a+1)(5+2) = 3a + 2(-2) + a$$

$$2(5a+2a+5+2) = 4a-4$$

$$14a+14 = 4a-4$$

$$10a = -18 \rightarrow a = -18/10 = \boxed{-9/5}$$

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$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$$

$$f(\overbrace{2-\sqrt{3}}^{x_1}) + f(\overbrace{2+\sqrt{3}}^{x_2}) = ?$$

x_1, x_2 معکوس هم اند

$$\sqrt{x_1} + \overbrace{\frac{1}{\sqrt{x_1}}}^{\sqrt{x_2}} + 2 + \sqrt{x_2} + \overbrace{\frac{1}{\sqrt{x_2}}}^{\sqrt{x_1}} + 2 = 4 + 2\sqrt{x_1} + 2\sqrt{x_2}$$

$$4 + 2\sqrt{2-\sqrt{3}} + 2\sqrt{2+\sqrt{3}} =$$

$$4 + 2(\overbrace{\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}}^{A}) \quad , A^2 = 6 \rightarrow \boxed{4+2\sqrt{6}}$$

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$$2f(x) - 3f(-x) = 4x^2 - x$$

$$(2f(x) - 3f(-x) = 4x^2 - x) \times 2$$

$$(2f(-x) - 3f(x) = 4x^2 + x) \times 3$$

$$4f(x) - 6f(-x) = 8x^2 - 2x$$

$$6f(-x) - 9f(x) = 12x^2 + 3x$$

$$-5f(x) = 20x^2 + x$$

$$f(x) = \frac{20x^2+x}{-5} = \boxed{-4x^2 - \frac{x}{5}}$$

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$$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$$

$f(0) = ?$

$$x=0 \rightarrow 2f(0) = 3m-1$$

$$f(0) = \frac{3m-1}{2}$$

$$x=-2 \rightarrow 6f(0) = 16+2m+3m-1 \rightarrow f(0) = \frac{5m+15}{6}$$

$$\frac{3m-1}{2} = \frac{5m+15}{6} \rightarrow m = 36/8 = 9/2$$

$$f(0) = \frac{3 \times \frac{9}{2} - 1}{2} = \boxed{\frac{25}{4}}$$

f خطی است $\rightarrow ax+b$

$$f(x)+f\left(\frac{1}{x}\right)=\frac{3x^2-12x+3}{x} \rightarrow x=-1 \rightarrow 2f(-1)=\frac{3+12+3}{-1}$$

$$2f(-1)=-18 \rightarrow \boxed{f(-1)=-9}$$