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نام و نام خانوادگی ....دینا بیاداریت بصیری........ پاسخنامه تشریحی تکلیف شماره ......۴.. کلاس یازدهم ریاضی ...پنجم ....

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الف) $f(x)=\sqrt{\dfrac{x-1}{x}-\dfrac{x}{x-1}} \Rightarrow \dfrac{x-1}{x}-\dfrac{x}{x-1}\geq 0 \Rightarrow \dfrac{x^2+1-2x-x^2}{(x)(x-1)}\geq 0 \Rightarrow \dfrac{1-2x}{(x)(x-1)}\geq 0$

$x=\frac{1}{2}$ ، $x=0$ ، $x=1$

$\to D_f=(-\infty,0)\cup[\frac{1}{2},1)$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \Rightarrow \begin{cases} x\neq -1 \\ x\neq 0 \\ x\neq 1 \\ x\neq -2 \\ \frac{3}{x-1}+\frac{1}{x+2}\neq 0 \Rightarrow \frac{3x+6+x-1}{(x-1)(x+2)}\neq 0 \Rightarrow \frac{4x+5}{(x-1)(x+2)}\neq 0 \end{cases}$

$x\neq -\frac{5}{4}$ ، $x\neq 1$ ، $x\neq -2$

$\cap\ D_f=\mathbb{R}-\{-2,-\frac{5}{4},-1,0,1\}$

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الف) $f(x)=\sqrt{\left((\frac{1}{3})^x-9\right)(32-4^x)} \Rightarrow \left((\frac{1}{3})^x-9\right)(32-4^x)\geq 0 \to$

$(\frac{1}{3})^x=3^2 \Rightarrow x=-2$ ، $32=4^x$ ، $2^5=2^{2x} \Rightarrow x=\frac{5}{2}$

$\to D_f=(-\infty,-2]\cup[\frac{5}{2},+\infty)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$\sqrt{y+1}=3-\sqrt[4]{x-1} \to \begin{cases} 3-\sqrt[4]{x-1}\geq 0 \to 81\geq x-1 \to 82\geq x \\ x-1\geq 0 \to x\geq 1 \end{cases}$ ، $3\geq\sqrt[4]{x-1}$ به توان ۴

$\cap\ D_f=[1,82]$

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$f(x)=\dfrac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1} \Rightarrow \begin{cases} x^2-x-2>0 \to (x-2)(x+1)>0 \to D=(-\infty,-1)\cup(2,+\infty) \\ \sqrt{x^2-1}+1\neq 0 \to \sqrt{x^2-1}\neq -1\ ✓ \\ x^2-1\geq 0 \to (x-1)(x+1)\geq 0 \Rightarrow D=(-\infty,-1]\cup[1,+\infty) \end{cases}$

$\cap\ D_f=(-\infty,-1)\cup(2,+\infty)$

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$\sqrt{3+ax-x^2}$ دامنه $[-2,b]$

$x=-2$ به ازای $\to 3-2a-4=0 \to 2a=-1 \to a=-\frac{1}{2}$

$f(x)=\sqrt{-x^2-\frac{1}{2}x+3}$ ریشه‌ها $x=\dfrac{-b\pm\sqrt{\Delta}}{2a}$ ، $(\frac{-1}{2})^2-4(3)(-1)$

$x_1=\dfrac{\frac{1}{2}+\sqrt{\frac{1}{4}+12}}{-2}=\dfrac{\frac{1}{2}+\frac{7}{2}}{-2}=\dfrac{+4}{-2}=-2$

$x_2=\dfrac{\frac{1}{2}-\sqrt{\frac{49}{4}}}{-2}=\dfrac{\frac{-6}{2}}{-2}=\frac{3}{2}\to b=\frac{3}{2}$

$a+b=-\frac{1}{2}+\frac{3}{2}=1$

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$f(x)=\begin{cases} 3x-2 & ; x\geq 1 \\ 2x+3 & ; x<1 \end{cases}$

$g(x)=\sqrt{f(x)-x} \Rightarrow f(x)-x\geq 0 \to f(x)\geq x$

$f(x)\geq x \to \begin{cases} x\geq 1 \to 3x-2\geq x \to 2x\geq 2 \to x\geq 1 \\ x<1 \to 2x+3\geq x \to x\geq -3 \end{cases}$ $\cap\ D_g=[-3,+\infty)$

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$$f(x) = \begin{cases} (a+1)(x+2) & ; x \geq 1 \\ 3a+2x & ; x < 1 \end{cases} \qquad 2f(5) = f(-2)+9 \to 2(7a+7) = 3a-4+9$$

$$\Leftrightarrow 10a = -18 \to a = \frac{-18}{10} = \frac{-9}{5}$$

$$f(5) = (a+1)(7) = 7a+7$$

$$f(-2) = 3a-4$$

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$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \to f(x) - 2 = \sqrt{x} + \frac{1}{\sqrt{x}} \xrightarrow{\text{به توان ۲}} (f(x)-2)^2 = x + \frac{1}{x} + 2$$

$$(f(2-\sqrt{3}) - 2)^2 = 2-\sqrt{3} + \frac{1 \times (2+\sqrt{3})}{(2-\sqrt{3}) \times (2+\sqrt{3})} + 2 = 2-\sqrt{3}+2+\sqrt{3}+2 = 6 \xrightarrow[\text{مثبت}]{} f(2-\sqrt{3})-2 = \sqrt{6} \Rightarrow f(2-\sqrt{3}) = \sqrt{6}+2$$

$$(f(2+\sqrt{3}) - 2)^2 = 2+\sqrt{3} + \frac{1 \times (2-\sqrt{3})}{(2+\sqrt{3}) \times (2-\sqrt{3})} + 2 = 2+\sqrt{3}+2-\sqrt{3}+2 = 6 \xrightarrow[\text{مثبت}]{} f(2+\sqrt{3})-2 = \sqrt{6} \to f(2+\sqrt{3}) = \sqrt{6}+2$$

$$f(2+\sqrt{3}) + f(2-\sqrt{3}) = \sqrt{6}+2+\sqrt{6}+2 = 4+2\sqrt{6}$$

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$$2f(x) - 3f(-x) = 4x^2 - x \xrightarrow{\times 2} 4f(x) - 6f(-x) = 8x^2 - 2x$$

$$2f(-x) - 3f(x) = 4x^2 + x \xrightarrow{\times 3} -9f(x) + 6f(-x) = 12x^2 + 3x$$

$$-5f(x) = 20x^2 + x \to f(x) = \frac{20x^2+x}{-5} = -4x^2 - \frac{1}{5}x$$

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$$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$$

$$x=-2 \to 6f(0) = 16+2m+3m-1 = 5m+15$$

$$x=0 \to 2f(0) = 3m-1 \xrightarrow{\times 3} 6f(0) = 9m-3$$

$$\to 9m-3 = 5m+15 \Rightarrow 4m = 18 \to m = \frac{9}{2}$$

$$2f(0) = \frac{27}{2} - 1 = \frac{25}{2} \to f(0) = \frac{25}{4}$$

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$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2-12x+3}{x} \to ax+b+\frac{a}{x}+b = 3x-12+\frac{3}{x} \Longrightarrow \begin{cases} a=3 \\ 2b=-12 \to b=-6 \end{cases}$$

$$f(x) = ax+b \longrightarrow f(1) = 3(1) - 6 = -9$$

$$f\left(\frac{1}{x}\right) = \frac{a}{x}+b$$

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