Subject: ۴

Year. Month. Date. ( )

دریا دیانی دردسمی

۱

1) $f(x)=\sqrt{\dfrac{\frac{x-1}{x}}{x-1}}$ … $\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}$

$\frac{x-1}{x}-\frac{x}{x-1}\geq 0 \to \frac{x^2+1-2x-x^2}{x^2-x} \to \frac{-2x+1}{x(x-1)}\geq 0$

0	$\frac{1}{2}$	1
+ ∄	− 0	+ ∄ −

$\to D_f=(-\infty,0)\cup[\frac{1}{2},1)$

2) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$

$\frac{x-2x-2}{x(x+1)} \to x(x+1)\neq 0 \to x\neq 0,-1$

$\frac{3x+6+x-1}{(x-1)(x+2)}$: $3x+6+x-1 \to 4x+5\neq 0 \to x\neq -\frac{5}{4}$; $(x-1)(x+2)\to x\neq 1,-2$

$D_f=\mathbb{R}-\{-2,-\frac{5}{4},1,-1,0\}$

۲

1) $\sqrt{\left((\frac{1}{3})^x-9\right)\left(32-4^x\right)}\geq 0$

$(\frac{1}{3})^x=9\to(\frac{1}{3})^x=3^2\to x=-2$

$32-4^x\to 32=2^{2x}\to 2^5=2^{2x}\to x=\frac{5}{2}$

−2	2.5
+ 0 −	0 +

$D_f=(-\infty,-2]\cup[2.5,+\infty)$

2) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$\sqrt{y+1}=3-\sqrt[4]{x-1} \to x-1\geq 0\to x\geq 1$

$3-\sqrt[4]{x-1}\geq 0\to \sqrt[4]{x-1}\leq 3$

$x-1\leq 81\to x\leq 82$

$D_f=[1,82]$

۳

$f(x)=\dfrac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+4} \to$ همواره بزرگتر از صفر

$-1 \quad 0 \quad 1 \quad 2$

$x^2-1\geq 0 \to$

−1	1
+ 0 −	0 +

$D_f=(-\infty,-1)\cup(2,+\infty)$

$x^2-x-2>0\to(x-2)(x+1)>0$

−1	2
+ 0 −	0 +

۴

$f(x)=\sqrt{3+ax-x^2}$ $\qquad x^2-ax-3\leq 0\to(x+2)(x-b)$

$D_f=[-2,b]$ $\qquad -2\times b=-3\to b=\frac{3}{2}$

$3-2a-4=0\to a=-\frac{1}{2}$ $\qquad \boxed{\frac{3}{2}-\frac{1}{2}=1=a+b}$

۵

$f(x)=\begin{cases}3x-2 & x\geq 1 \to\\ 2x+3 & x<1\end{cases}$

$g(x)=\sqrt{f(x)-x}$

1) $f(x)-x\to x\geq 1\to 3x-2-x=2x-2\geq 0\to x\geq 1$

2) $f(x)-x\to x<1\to 2x+3-x=x+3\geq 0\to x\geq -3$

$\Big\}\ D_f=[-3,+\infty)$

(6)

$$f(x)=\begin{cases}(a+1)(x-2) & x\geq 1\\ 3a+2x & x<1\end{cases}$$

$$2f(5)=f(-2)+a \rightarrow 2(a+1)(5+2)=3a+2(-2)+a$$

$$14a+14=4a-4 \rightarrow 10a=-18 \rightarrow \boxed{a=-1.8}$$

(7)

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2$$

$$f(2-\sqrt{3})+f(2+\sqrt{3}) \qquad \sqrt{x_1}+\frac{1}{\sqrt{x_1}}+2+\sqrt{x_2}+\frac{1}{\sqrt{x_2}}+2=4+2\sqrt{x_1}+2\sqrt{x_2}$$

معکوس هم هستند چون ضربشان یک میشود!

$$4+\underbrace{2\sqrt{2-\sqrt{3}}+2\sqrt{2+\sqrt{3}}}_{2(\underbrace{\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}}_{A})}=\boxed{4+2\sqrt{6}}$$

$$A^2=2-\sqrt{3}+2+\sqrt{3}+2=6 \rightarrow A=\sqrt{6}$$

(8)

$$2f(x)-3f(-x)=4x^2-x$$

$$f(x)=?$$

$$\begin{cases}(2f(x)-3f(-x)=4x^2-x)\times 2\\ (2f(-x)-3f(x)=4x^2+x)\times 3\end{cases}$$

$$4f(x)-6f(-x)=8x^2-2x$$

$$6f(-x)-9f(x)=12x^2+3x$$

$$-5f(x)=20x^2+x \rightarrow \boxed{f(x)=-4x^2-\frac{x}{5}}$$

(9)

$$(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1$$

$$f(0)=?$$

$$1)\ x=0 \rightarrow (2f(0)=3m-1)\xrightarrow{\times 3} 6f(0)=9m-3$$

$$2)\ x=-2 \rightarrow 6f(0)=16+5m-1=15+5m$$

$$9m-3=15+5m \rightarrow 4m=18 \rightarrow \boxed{m=4.5}$$

$$2f(0)=3\times 4.5-1=12.5 \rightarrow \boxed{f(0)=6.25}$$

(10)

f خطی است $\rightarrow ax+b$

$$f(x)+f\left(\frac{1}{x}\right)=\frac{3x^2-12x+3}{x}$$

$$f(-1)=\ \rightarrow 2f(-1)=\frac{3+12+3}{-1}=-18 \rightarrow 2f(-1)=-18 \rightarrow \boxed{f(-1)=-9}$$