Subject: ۴
Year. Month. Date. ( )

۲۰

دریا دیانی دردشتی

(۱)

1) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}$

$\frac{x-1}{x}-\frac{x}{x-1}\geq 0 \to \frac{x^2+1-2x-x^2}{x^2-x}\to \frac{-2x+1}{x(x-1)}\geq 0$

$0 \quad \frac{1}{2} \quad 1$ ; $+ \; - \; + \; -$ $\to D_f=(-\infty,0)\cup[\frac{1}{2},1)$

2) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$

$\frac{x-2x-2}{x(x+1)} \to x(x+1)\neq 0 \to x\neq 0,-1$

$\frac{3x+6+x-1}{(x-1)(x+2)} \to 4x+5\neq 0 \to x\neq -\frac{5}{4}$

$(x-1)(x+2)\to x\neq 1,-2$

$D_f=\mathbb{R}-\{-2,-\frac{5}{4},1,-1,0\}$

(۲)

1) $\sqrt{((\frac{1}{3})^x-9)(32-4^x)}\geq 0$

$(\frac{1}{3})^x=9\to(\frac{1}{3})^x=3^2\to x=-2$

$32-4^x\to 32=2^{2x}\to 2^5=2^{2x}\to x=\frac{5}{2}$

$-2 \quad 2.5$ ; $+ \; - \; +$ $\quad D_f=(-\infty,-2]\cup[2.5,+\infty)$

2) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$\sqrt{y+1}=3-\sqrt[4]{x-1} \to x-1\geq 0\to x\geq 1$

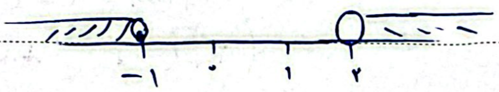
$3-\sqrt[4]{x-1}\geq 0 \to \sqrt[4]{x-1}\leq 3$

$x-1\leq 81 \to x\leq 82$

$D_f=[1,82]$

(۳)

$f(x)=\dfrac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+4}$ $\to$ همواره بزرگتر از صفر



$x^2-1\geq 0 \to$ $-1 \quad 1$ ; $+ \; - \; +$

$D_f=(-\infty,-1)\cup(2,+\infty)$

$x^2-x-2>0 \to (x-2)(x+1)>0$ $\quad -1 \quad 2$ ; $+ \; - \; +$

(۴)

$f(x)=\sqrt{3+ax-x^2}$ $\qquad x^2-ax-3\leq 0 \to (x+2)(x-b)$

$D_f=[-2,b]$ $\qquad -2\times b=-3 \to b=\frac{3}{2}$

$3-2a-4=0 \to a=-\frac{1}{2}$ $\qquad \frac{3}{2}-\frac{1}{2}=\boxed{1=a+b}$

(۵)

$f(x)=\begin{cases}3x-2 & x\geq 1 \to \\ 2x+3 & x<1\end{cases}$

$g(x)=\sqrt{f(x)-x}$

1) $f(x)-x \to x\geq 1 \to 3x-2-x=2x-2\geq 0 \to x\geq 1$

2) $f(x)-x \to x<1 \to 2x+3-x=x+3\geq 0 \to x\geq -3$

$\Big\} D_f=[-3,+\infty)$

۶

$$f(x)=\begin{cases}(a+1)(x-2) & x>1\\ 3a+2x & x\le 1\end{cases}$$

$2f(5)=f(-2)+a \rightarrow 2(a+1)(5+2)=3a+2(-2)+a$

$14a+14=4a-4 \longrightarrow 10a=-18 \longrightarrow \boxed{a=-1{,}8}$

۷

$f(x)=\sqrt{x}+\dfrac{1}{\sqrt{x}}+2$

$f(2-\sqrt{3})+f(2+\sqrt{3})$

معکوس هم هستند چون ضربشان یک شد!

$\sqrt{x_1}+\dfrac{1}{\sqrt{x_1}}+2+\sqrt{x_2}+\dfrac{1}{\sqrt{x_2}}+2=4+2\sqrt{x_1}+2\sqrt{x_2}$

$4+\underbrace{2\sqrt{2-\sqrt{3}}+2\sqrt{2+\sqrt{3}}}_{2(\underbrace{\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}}_{A})}=\boxed{4+2\sqrt{6}}$

$A^2=2-\sqrt{3}+2+\sqrt{3}+2=6 \rightarrow A=\sqrt{6}$

۸

$2f(x)-3f(-x)=4x^2-x$

$f(x)=?$

$(2f(x)-3f(-x)=4x^2-x)\times 2$

$(2f(-x)-3f(x)=4x^2+x)\times 3$

$4f(x)-6f(-x)=8x^2-2x$

$6f(-x)-9f(x)=12x^2+3x$

$-5f(x)=20x^2+x \longrightarrow \boxed{f(x)=-4x^2-\dfrac{x}{5}}$

۹

$(x+2)f(x)-2xf(x+2)=4x^2-mx+3m-1$

$f(0)=?$

$1)\ x=0 \longrightarrow (2f(0)=3m-1)\times 3 \rightarrow 6f(0)=9m-3$

$2)\ x=-2 \longrightarrow 4f(0)=16+5m-1=15+5m$

$9m-3=15+5m \longrightarrow 4m=18 \rightarrow \boxed{m=4{,}5}$

$2f(0)=3\times 4{,}5-1=12{,}5 \longrightarrow \boxed{f(0)=6{,}25}$

۱۰

$f$ خطی است $\rightarrow ax+b$

$f(x)+f\left(\dfrac{1}{x}\right)=\dfrac{3x^2-12x+3}{x}$

$f(-1)=\ \longrightarrow 2f(-1)=\dfrac{3+12+3}{-1}=-18 \longrightarrow 2f(-1)=-18 \rightarrow \boxed{f(-1)=-9}$