آمادگی ریاضی

۱) الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}$ $D=(-\infty,0)\cup[\frac{1}{2},1)$

$\frac{x-1}{x}-\frac{x}{x-1}\geq 0 \rightarrow \frac{x^2+1-2x-x^2}{x^2-x}=\frac{1-2x}{x(x-1)}\geq 0$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$ $x+1\neq 0\rightarrow x\neq -1$ $\frac{3}{x-1}+\frac{1}{x+2}\neq 0$

$x\neq 0$ $\rightarrow D=R-\{-2,-\frac{5}{4},-1,0,1\}$

$x-1\neq 0\rightarrow x\neq 1$

$x+2\neq 0\rightarrow x\neq -2$

---

۲) الف) $f(x)=\sqrt{((\frac{1}{3})^x-9)(32-4^x)}$

$((\frac{1}{3})^x-9)(32-4^x)\geq 0$ $(\frac{1}{3})^x-9=0\rightarrow(\frac{1}{3})^x=9\rightarrow(\frac{1}{3})^x=3^2\rightarrow 3^{-x}=3^2$

$32-4^x=0\rightarrow 2^5=4^x\rightarrow 2^5=2^{2x}\rightarrow x=\frac{5}{2}$

$D=(-\infty,-2]\cup[\frac{5}{2},+\infty)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$ $\sqrt{y+1}=3-\sqrt[4]{x-1}$ $3-\sqrt[4]{x-1}\geq 0$

$\rightarrow 3\geq\sqrt[4]{x-1}\rightarrow 81\geq x-1\rightarrow x\leq 82$ (I)

$\sqrt[4]{x-1}\xrightarrow{\text{زوج}} x-1\geq 0\rightarrow x\geq 1$ (II) $D_1\cap D_2=[1,82]$

---

۳) $f(x)=\dfrac{\log_2(x^2-x-2)}{\sqrt{x^2-1}+1}$ $x^2-x-2>0\rightarrow(x-2)(x+1)>0$

$\sqrt{x^2-1}+1\neq 0\rightarrow\sqrt{x^2-1}\neq -1$ $D_1=(-\infty,-1)\cup(2,+\infty)$

همیشه برقرار بود

$x^2-1\geq 0\rightarrow x^2\geq 1\rightarrow x\geq 1\cup x\leq -1$

$D_2=[1,+\infty)\cup(-\infty,-1]$ $D_1\cap D_2=(-\infty,-1)\cup(2,+\infty)$

$\sqrt{۳+ax-x^۲} \to D=[-۲, b] \qquad a+b=?$ (۲

$۳+ax-x^۲ \geq ۰$

$S=x_۱+x_۲=-\frac{b}{a} \to \frac{-a}{-۱} \to -۲+b=a \to \frac{۳}{۲}-۲=a \to a=-\frac{۱}{۲}$

$P=x_۱ x_۲=\frac{c}{a} \to -۲\times b=\frac{۳}{-۱} \to ۲b=۳ \to b=\frac{۳}{۲}$

$a+b=-\frac{۱}{۲}+\frac{۳}{۲}=\boxed{۱}$

$f(x)=\begin{cases} ۳x-۲ & x\geq ۱ \\ ۴x+۳ & x<۰ \end{cases}$ (۵

$x\geq ۱ \to ۳x-۲\geq x \to ۲x\geq ۲ \to x\geq ۱$

$۴x+۳\geq x \to x\geq -۱ \to -۱\leq x<۰$

$g(x)=\sqrt{f(x)-x} \to f(x)\geq x$

$D_{g(x)}=D_۱\cup D_۲=\underline{[-۱, +\infty)}$

$f(x)=\begin{cases} (a+۱)(x+۲) & x>۱ \\ ۳a+۲x & x\leq ۱ \end{cases}$ (۴

$۲f(۵)=f(-۲)+a \to ۲(۷a+۷)=۳a-۴+a \to ۱۴a+۱۴=۳a-۴+a$

$\to ۱۰a=-۱۸ \to a=\boxed{\frac{-۱۸}{۱۰}}$

$f(۵)=(a+۱)(۵+۲)=۷a+۷$

$f(-۲)=۳a-۴$

$f(x)=\sqrt{x}+\frac{۱}{\sqrt{x}}+۲ \qquad f(۲+\sqrt{۳})=\sqrt{۲+\sqrt{۳}}+\frac{۱}{\sqrt{۲+\sqrt{۳}}}+۲=\sqrt{۲+\sqrt{۳}}+\sqrt{۲-\sqrt{۳}}+۲$ (۷

$f(۲-\sqrt{۳})+f(۲+\sqrt{۳})=? \qquad \sqrt{۲-\sqrt{۳}}+\sqrt{۲+\sqrt{۳}}+۲+\sqrt{۲+\sqrt{۳}}+\sqrt{۲-\sqrt{۳}}+۲=۲\left(\sqrt{۲+\sqrt{۳}}+\sqrt{۲-\sqrt{۳}}\right)+۴=\boxed{۲\sqrt{۶}+۴}$

$\left(\sqrt{۲+\sqrt{۳}}+\sqrt{۲-\sqrt{۳}}\right)^۲=۲+\sqrt{۳}+۲-\sqrt{۳}+۲\sqrt{۴-۳}=۶ \to \underline{\sqrt{۲+\sqrt{۳}}+\sqrt{۲-\sqrt{۳}}=\sqrt{۶}}$

(8

$2f(x) - 3f(-x) = 4x^2 - x \qquad f(x) = ?$

$-x \rightarrow 2f(-x) - 3f(+x) = 4x^2 + x \xrightarrow{\times 3}$

$4f(x) - 6f(-x) = 8x^2 - 2x$

$9f(-x) - 9f(x) = 12x^2 + 3x$

$-5f(x) = 20x^2 + x \rightarrow f(x) = \dfrac{-20x^2 - x}{5} \rightarrow \boxed{-4x^2 - \frac{1}{5}x}$

(9

$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3x - 1$

$x = 0 \rightarrow 2f(0) = 3m - 1 \longrightarrow 4f(0) = \cancel{15+5m}\ 9m - 3$

$x = -2 \rightarrow 4f(0) = 16 + 2m + 3m - 1 \Rightarrow 15 + 5m \rightarrow 4f(0) = 15 + 5m$

$9m - 3 = 15 + 5m$

$\rightarrow 4m = 18 \rightarrow m = \dfrac{18}{4} = \boxed{\dfrac{9}{2}}$

$m = \frac{9}{2} \qquad 2f(0) = 3\left(\frac{9}{2}\right) - 1 = \frac{27}{2} - 1 = \frac{25}{2} \rightarrow f(0) = \dfrac{\frac{25}{2}}{2} = \dfrac{25}{4}$

(10

$f(x) + f\left(\frac{1}{x}\right) = \dfrac{3x^2 - 12x + 3}{x}$

$x = -1 \rightarrow f(-1) + f(-1) = \dfrac{3 + 12 + 3}{-1} \rightarrow 2f(-1) = -18 \rightarrow f(-1) = \boxed{-9}$