حل المسائل

تکلیف ۳:

۱- الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} = \sqrt{\frac{(x-1-x)(x-1+x)}{x(x-1)}}$

$= \sqrt{\frac{-1(2x-1)}{x(x-1)}}$

0	$\frac{1}{2}$	1
$+$ $-$	$+$	$-$

$D_{f(x)} = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{2}{x-1} + \frac{1}{x+2}} = \frac{\frac{x-2}{(x+1)x}}{4x+5} = \frac{x-2}{x(x+1)(4x+5)}$

$D_{f(x)} = \mathbb{R} - \{0, -1, -\frac{5}{4}\}$

۲- الف) $f(x) = \sqrt{\left((\frac{1}{3})^x - 9\right)\left(32 - 4^x\right)} \geq 0$

$(3^{-x} - 3^2)(2^5 - 2^{2x})$

$-2 \qquad \frac{5}{2}$: $- \quad + \quad -$

$-x = 2 \to x = -2 \qquad 2x = 5 \to x = \frac{5}{2}$

$D_{f(x)} = [-2, \frac{5}{2}]$

ب) $\sqrt[4]{x-1} + \sqrt{y+1} = 3$

$\sqrt[4]{x-1} - 3 = -\sqrt{y+1} \xrightarrow{\text{به توان ۲}} (\sqrt{x-1}) + 9$

$x - 1 \geq 0 \to x \geq 1$

$-6\sqrt{x-1} = y+1$

$y = -6\sqrt{x-1} - 1$

$x \geq 1$

$D_{f(x)} = [1, +\infty)$

۳- $f(x) = \frac{\log_4 (x^2 - x - 2)}{\sqrt{x^2-1} + 1}$

$x^2 - x - 2 > 0 \to (x-2)(x+1) > 0$

$-1 \qquad 2$: $+ \quad - \quad +$

$x = (-\infty, -1) \cup (2, +\infty)$ $\cap$

$x^2 - 1 \geq 0$

$1 \leq x^2 \to x \leq -1 \cup 1 \leq x$

$D_{f(x)} = (-\infty, -1) \cup (2, +\infty)$

حل تمرین بخش

$f(x) = f\left(\frac{1}{x}\right) \rightarrow \frac{2x^2 - 12x + 2}{x}$ _45

$f_{(x)} = ax + b \rightarrow f(-1) \rightarrow f(-1) + f(-1) = \frac{2(-1)^2 + 12 + 2}{-1} = -18$

$2f(-1) = -18 \rightarrow f(-1) = -9$

$x = 0 \rightarrow f(2)\, f(0) = 3m - 1$ _4

$x = -2 \rightarrow f(0)\, f(-2) + 6f(0) = 16 + 2m + \underbrace{3m - 1}_{f(2)\, f(0)}$

$f(0)\, f(-2) - f(2)\, f(0) + 6f(0) = 16 + 2m$

$f(0)\big(f(\underbrace{-2}_{t}) - f(\underbrace{2}_{-t})\big) = f(0)\, 16m - 6f(0) = 16 + 2m$

$f(0)(16m - 6) = 16 + 2m \rightarrow f(0) = \frac{16 + 2m}{16m - 6}$

$\left.\begin{aligned} t &= 3t^2 - mt + 3m - 1 \\ -t &= 3t^2 + mt + 3m - 1 \end{aligned}\right\} -8mt = 16m$

حل تمرین تابع ها

۴- $f(x)=\sqrt{2+ax-x^2}$ $[-2, b]$

$\frac{c}{a}=\frac{2}{-1}=-2b \rightarrow b=\frac{2}{2}$ ضرب ریشه‌ها

جمع ریشه‌ها $=-\frac{b}{a}=-\frac{a}{2}$

$2+ax-x^2\geq 0 \rightarrow x=-2 \rightarrow -2a-2\geq 0 \rightarrow a\leq -1$

$b \quad x=\frac{3}{2} \rightarrow 2+\frac{3}{2}a-\frac{9}{4}\geq 0 \rightarrow \frac{3}{2}a\geq \frac{1}{4} \rightarrow a\geq \frac{1}{6}$

۵- $f(x)-x=3x-2-x=2x-2 \quad x\geq 1 \quad [0,+\infty)$

$2x+2-x=x+2 \quad x<1 \quad [0,3)$

$x+2=0 \rightarrow x=-2$

$f(x)-x\geq 0 \longrightarrow D_{f(x)}=[-2,+\infty)$

۶- $f(x)=\begin{cases}(a+1)(x+2) & x>1\\ 2a+x & x\leq 1\end{cases}$ $\quad 2f(a)=f(-2)+a$

$f(5)=(a+1)(7)=7a+7 \rightarrow 2f(a)=14a+14$

$14a+14=2a-4+a$

$10a=-18$

$a=-1/8$

$4=2+2=f(2-\sqrt{3})+f(2+\sqrt{3})=?\ f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2-7$

$f(x)=2$

$\sqrt{x}+\frac{1}{\sqrt{x}}=1$ عبارت همواره بزرگتر از ۲

۸- $2\times\left(2f(x)-3f(-x)=4x^2-x\right) \rightarrow 4f(x)-6f(-x)=8x^2-2x$

$3\times\left(2f(-x)-3f(x)=4(-x)^2-(-x)\right) \rightarrow 6f(-x)-9f(x)=12x^2+3x$

$4x^2+x$

$-5f(x)=20x^2+x$

$f(x)=-4x^2-\frac{x}{5}$