حلیمه اسعدیان

تکلیف ۳ : ۱۵٫۲۵

۱- الف) $f(x) = \sqrt{\dfrac{x-1}{x} - \dfrac{x}{x-1}} = \sqrt{\dfrac{(x-1)^2 - x^2}{x(x-1)}} = \sqrt{\dfrac{(x-1-x)(x-1+x)}{x(x-1)}}$

$= \sqrt{\dfrac{-1(2x-1)}{x(x-1)}}$

$\begin{array}{ccccccc} & 0 & & \frac{1}{2} & & 1 & \\ + & & - & & + & & - \end{array}$

$D_{f(x)} = (-\infty, 0) \cup [\frac{1}{2}, 1)$

۱٫۵

ب) $f(x) = \dfrac{\dfrac{1}{x+1} - \dfrac{2}{x}}{\dfrac{2}{x-1} + \dfrac{1}{x+2}} = \dfrac{\dfrac{x-2}{(x+1)x}}{4x+5} = \dfrac{x-2}{x(x+1)(4x+5)}$

$D_{f(x)} = \mathbb{R} - \{0, -1, -\frac{5}{4}\}$

$x - 1 \neq 0 \to x \neq 1$

$x + 2 \neq 0 \to x \neq -2$

۲- الف) $f(x) = \sqrt{\left(\left(\frac{1}{3}\right)^x - 9\right)\left(32 - 4^x\right)} \geq 0$

$(3^{-x} - 3^2)(2^5 - 2^{2x})$

$\begin{array}{ccccccc} & -2 & & 2.5 & & \\ - & & + & & - \end{array}$

$\begin{array}{ccc} + & - & + \end{array}$

۱

$-x = 2 \to x = -2$ $\qquad 2x = 5 \to x = 2.5$

$D_f: (-\infty, -2] \cup [2.5, +\infty)$

$D_{f(x)} = [-2, 2.5]$

۲- ب)

$x - 1 \geq 0 \to x \geq 1$

$3 - \underbrace{\sqrt[4]{x-1}}_{\geq 0} = \underbrace{\sqrt{y+1}}_{+} \to x \leq 82 \quad D_f = [1, 82]$

ب) $\sqrt[4]{x-1} + \sqrt{y+1} = 2$ $\qquad (\sqrt[4]{x-1} - 2) = (-\sqrt{y+1})^2 \xrightarrow{\text{به توان ۲ می‌رسانیم}} (\sqrt{x-1}) + 4$

$x - 1 \geq 0 \to x \geq 1$ $\qquad -4\sqrt[4]{x-1} = y + 1$

$y = -4\underbrace{\sqrt[4]{x-1}}_{x \geq 1} - 1$

$D_{f(x)} = [1, +\infty)$

۳- $f(x) = \dfrac{\log_4 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$ $\qquad x^2 - x - 2 > 0 \to (x-2)(x+1) > 0$

$\begin{array}{ccccccc} & -1 & & 2 & & \\ + & & - & & + \end{array}$

۲

$x^2 - 1 \geq 0$ $\qquad$ $x = (-\infty, -1) \cup (2, +\infty)$ اشتراک

$1 \leq x^2 \longrightarrow x \leq -1 \cup 1 \leq x$

$D_{f(x)} = (-\infty, -1) \cup (2, +\infty)$

حل تمرین پنجم

$f(x) + f(\frac{1}{x}) = \frac{4x^2 - 12x + 2}{x}$ —۹۵

$f(x) = ax + b \rightarrow f(-1) \rightarrow f(-1) + f(-1) = \frac{4(-1)^2 + 12 + 2}{-1} = -18$

$2f(-1) = -18 \rightarrow f(-1) = -9$

$x = 0 \rightarrow \cancel{f(2)}\, 2f(0) = 3m - 1 \rightarrow f(0) = \frac{3m-1}{2}$ —۹

$x = -2 \rightarrow f(0)\, \cancel{f(-2)} + 4f(0) = 16 + 2m + 3m - 1$ $\quad f(0) = \frac{18 + 5m}{2}$

$f(2)\, f(0)$

$f(0) f(-2) - f(2) f(0) + 4f(0) = 14 + 2m$ $\quad m = \frac{9}{2} \quad f(0) = \frac{25}{2}$

$f(0)\big(f(\underbrace{-2}_{t}) - f(\underbrace{2}_{-t})\big) = f(0)\, 16m - 4f(0) = 14 + 2m$

$f(0)(16m - 4) = 14 + 2m \rightarrow f(0) = \frac{14 + 2m}{16m - 4}$

$\left.\begin{array}{l} t = 4t^2 - mt + 3m - 1 \\ -t = 4t^2 + mt + 3m - 1 \end{array}\right\} -8mt = 14m$

حل تمرین فصل ۲ تابع ها

۴- $f(x)=\sqrt{2+ax-x^2}$ $[-2,b]$

مجموع ریشه ها $= -\frac{b}{a} = -\frac{a}{-1} = -2+b$ ~~۰٫۵~~

۱٫۷۵

$-1/5$

ضرب ریشه ها $\frac{c}{a} = \frac{2}{-1} = -2b \rightarrow b = \frac{3}{2} = 1/5$

$a = \frac{1}{2}$

$a+b=1$

$2+ax-x^2 \geq 0 \rightarrow x=-2 \rightarrow -2a-1 \geq 0 \rightarrow a \leq \frac{1}{2}$

$b\ \ x=\frac{3}{2} \rightarrow 2+\frac{3}{2}a-\frac{9}{4} \geq 0 \rightarrow \frac{3}{2}a \geq \frac{1}{4} \rightarrow a \geq \frac{1}{6}$

۵- $f(x)-x = 3x-2-x = 2x-2$ $x \geq 1$ $[0,+\infty)$

$2x+2-x = x+2$ $x<1$ $[0,3)$

$x+2=0 \rightarrow x=-2$

۲

$f(x)-x \geq 0 \longrightarrow D_{f(x)} = [-2,+\infty)$

۶- $f(x)=\begin{cases} (a+1)(x+2) & x>1 \\ 2a+3x & x \leq 1 \end{cases}$ $2f(5) = f(-2)+a$

$f(5) = (a+1)(7) = 7a+7 \rightarrow 2f(5) = 14a+14$

۲

$14a+14 = 2a-4+a$

$10a = -18$

$f(-2) = 2a-4$

$a = -1/8$

۷- $\sqrt{x}+\frac{1}{\sqrt{x}}+2$ جواب $f(x)$ ؟ $= f(2-\sqrt{2})+f(2+\sqrt{2}) = 2+2 = 6$

$f(x)=2$ $(2-\sqrt{2})(2+\sqrt{2})=1$ $\sqrt{x}$ و $\frac{1}{\sqrt{x}}$ عکس همدیگرند $=1$

عکس همدیگر

۵

جواب آخر $= 2\sqrt{6}+2$

۸- $2x^2-2x$ $\rightarrow 6f(-x) + 8 \ \ \ 4f(x)$ $-$ $(2f(x) - 3f(-x) = 4x^2-x)$ ۳×

$+$

۲ $\times (2f(-x) - 3f(x) = 2(-x)^2-(-x)) \rightarrow$ $6f(-x) - 9f(x) = 12x^2 - 3x$

$4x^2+x$

۲

$-5f(x) = 20x^2+x$

$f(x) = -4x^2-\frac{x}{5}$