نام و نام خانوادگی: آیلین عظیمی   تکلیف: ۴   کلاس: یازدهم ریاضی

۱

الف)

$$f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \rightsquigarrow$$

① $\frac{x-1}{x}-\frac{x}{x-1}\geq 0 \to \frac{\cancel{x^2}+1-2x-\cancel{x^2}}{x(x-1)}=\frac{1-2x}{x(x-1)}\geq 0$

② $x\neq 0$

③ $x-1\neq 0 \rightsquigarrow x\neq 1$

x		0		$\frac{1}{2}$		1	
	$+$		$-$		$+$		$-$

$\boxed{D_f: (-\infty,0)\cup[\frac{1}{2},1)}$

ب)

$$f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \rightsquigarrow$$

$x+1\neq 0 \to x\neq -1$

$x\neq 0$

$x-1\neq 0 \to x\neq 1$

$x+2\neq 0 \to x\neq -2$

$\frac{3}{x-1}+\frac{1}{x+2}\neq 0 \to \frac{3x+6+x-1}{(x-1)(x+2)}=\frac{4x+5}{(x-1)(x+2)}\neq 0$

$4x+5\neq 0$

$x\neq \frac{-5}{4}$

$\boxed{D_f: \mathbb{R}-\{-1,0,1,-2,\frac{-5}{4}\}}$

۲

الف)

$$f(x)=\sqrt{\left(\left(\tfrac{1}{3}\right)^x-9\right)\left(32-4^x\right)} \to (3^{-x}-9)(32-2^{2x})\geq 0$$

$x=-2$ ، $x=\frac{5}{2}$

x		-2		$\frac{5}{2}$	
	$+$		$-$		$+$

$\boxed{D_f: (-\infty,-2]\cup[\frac{5}{2},+\infty)}$

ب)

$$\sqrt[4]{x-1}+\sqrt{y+1}=3$$

① $x-1\geq 0 \to x\geq 1$

② $\sqrt{y+1}=3-\sqrt[4]{x-1} \to 3-\sqrt[4]{x-1}\geq 0 \to \sqrt[4]{x-1}\leq 3 \xrightarrow{()^4} x-1\leq 81$

$x\leq 82$

$\boxed{D_f: [1,82]}$

۳

$$f(x)=\frac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1} \rightsquigarrow$$

① $x^2-x-2>0 \to (x-2)(x+1)>0$

x		-1		2	
	$+$		$-$		$+$

$x: (-\infty,-1)\cup(2,+\infty)$

② $x^2-1\geq 0 \to x^2\geq 1 \rightsquigarrow x\geq 1$ $\cup$ $x\leq -1$

$\Rightarrow \boxed{D_f: (-\infty,-1)\cup(2,+\infty)}$

۴

$$f(x)=\sqrt{3+ax-x^2} \to -x^2+ax+3\geq 0$$

$D_f: [-2,b]$

$P=ax^2+bx+c$

x	
P	موافق a

x		x_1	
P	مخالف a		موافق a

x		x_1		x_2	
P	موافق a		مخالف a		موافق a
	$-$		$+$		$-$

D_f

x		-2		b	
	$-$		$+$		$-$

$f(-2)=0 \to -4-2a+3=0$

$2a=-1$

$\boxed{a=\frac{-1}{2}}$

$f(b)=0 \to -b^2+ab+3=0$

$-b^2-\frac{1}{2}b+3=0$

$b^2+\frac{1}{2}b-3=0$

$2b^2+b-6=0$

$b^2+b-12=0$

$(b+\frac{4}{2})(2b-3)=12 \to (b+2)(2b-3)=0$

$b=-2$ ✗

$\boxed{b=\frac{3}{2}}$

$$a+b=\frac{-1}{2}+\frac{3}{2}=1$$

۵

$$f(x)\begin{cases}3x-2 & x\geq 1\\ 2x+3 & x<1\end{cases}$$

$g(x)=\sqrt{f(x)-x} \to f(x)-x\geq 0 \to f(x)\geq x$

$$\begin{cases}3x-2\geq x \quad x\geq 1 \to 2x-2\geq 0 \quad x\geq 1 ✓ \\ \quad x\geq 1 \\ \cup \\ 2x+3\geq x \quad x<1 \to x+3\geq 0 \quad x<1 \\ \quad x\geq -3\end{cases}$$

$x: [-3,1)$

$\boxed{D_f: [-3,+\infty)}$

CamScanner

$$f(x)\begin{cases}(a+1)(x+2) & x>1\\ 3a+2x & x\le 1\end{cases}$$

$$2f(5)=f(-2)+a \rightsquigarrow f(5)=(a+1)7 \quad f(-2)=3a-4 \Rightarrow 2\times7(a+1)=3a-4+a$$

$$14a+14=4a-4$$

$$10a=-18$$

$$\boxed{a=-1.8}$$ 6

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2$$

$$2-\sqrt{3}=\left(\sqrt{\tfrac{1}{2}}-\sqrt{\tfrac{3}{2}}\right)^2$$

$$2+\sqrt{3}=\left(\sqrt{\tfrac{1}{2}}+\sqrt{\tfrac{3}{2}}\right)^2$$

$$f(2-\sqrt{3})+f(2+\sqrt{3})=\sqrt{2-\sqrt{3}}+\frac{1}{\sqrt{2-\sqrt{3}}}+2+\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}+2=\sqrt{\tfrac{3}{2}}-\cancel{\sqrt{\tfrac{1}{2}}}+\frac{1}{\sqrt{\tfrac{3}{2}}-\sqrt{\tfrac{1}{2}}}+\sqrt{\tfrac{3}{2}}+\cancel{\sqrt{\tfrac{1}{2}}}+\frac{1}{\sqrt{\tfrac{3}{2}}+\sqrt{\tfrac{1}{2}}}+\dots$$

$$\left|\sqrt{\tfrac{1}{2}}-\sqrt{\tfrac{3}{2}}\right|=\sqrt{\tfrac{3}{2}}-\sqrt{\tfrac{1}{2}} \qquad \sqrt{\tfrac{1}{2}}+\sqrt{\tfrac{3}{2}} \qquad \frac{\sqrt{3}-1}{\sqrt{2}} \qquad \frac{\sqrt{3}+1}{\sqrt{2}}$$

$$=2\sqrt{\tfrac{3}{2}}+\underbrace{\frac{\sqrt{2}}{\sqrt{3}-1}+\frac{\sqrt{2}}{\sqrt{3}+1}}_{\text{مزدوج}}+4=2\sqrt{\tfrac{3}{2}}+\frac{\sqrt{6}+\cancel{\sqrt{2}}+\sqrt{6}-\cancel{\sqrt{2}}}{2}+4=\frac{2\sqrt{3}}{\sqrt{2}}+\sqrt{6}+4=\boxed{2\sqrt{6}+4}$$

$$(2f(x)-3f(-x)=4x^2-x)\times 2$$

$$(-3f(x)+2f(-x)=4x^2+x)\times 3$$

$$\rightsquigarrow \quad 4f(x)-6\cancel{f(-x)}=8x^2-2x$$

$$-9f(x)+\cancel{6f(-x)}=12x^2+3x$$

$$-5f(x)=20x^2+x$$

$$\boxed{f(x)=-4x^2-0.2x}$$ 8

$$(x+2)f(x)-3x\,f(x+2)=4x^2-mx+3m-1$$

$$x=0 \rightsquigarrow 2f(0)=3m-1 \rightarrow f(0)=\frac{3m-1}{2}$$

$$x=-2 \rightsquigarrow 6f(0)=16+2m+3m-1=15+5m \rightarrow f(0)=\frac{5m+15}{6}$$

$$\frac{3m-1}{2}=\frac{5m+15}{6}$$

$$10m+30=18m-6$$

$$8m=36$$

$$m=\frac{36}{8}=\frac{9}{2}$$ 9

$$f(0)=\frac{\frac{27}{2}-1}{2}=\boxed{\frac{25}{4}}$$

$$f(x)+f\left(\frac{1}{x}\right)=\frac{3x^2-12x+3}{x} \rightsquigarrow ax+b+\frac{a}{x}+b=ax+\frac{a}{x}+2b=3x+\frac{3}{x}-12$$

$$f(x)=ax+b \qquad a=3,\quad b=-6 \rightarrow y=3x-6$$ 10

$$f(-1) \Rightarrow y=-3-6=-9$$