نام و نام خانوادگی: آیلین عظیمی   تصحیح: ۴   کلاس: تجربی یازدهم

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الف) $f(x)=\sqrt{\dfrac{x-1}{x}-\dfrac{x}{x-1}}$ ⇝

(۱) $\dfrac{x-1}{x}-\dfrac{x}{x-1}\geq 0 \rightarrow \dfrac{\cancel{x^2}+1-2x-\cancel{x^2}}{x(x-1)}=\dfrac{1-2x}{x(x-1)}\geq 0$

(۲) $x\neq 0$

(۳) $x-1\neq 0 \rightsquigarrow x\neq 1$

x		0		$\frac{1}{2}$		1	
	$+$		$-$		$+$		$-$

$\boxed{D_f: (-\infty,0)\cup[\frac{1}{2},1)}$

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ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$ ⇝

$x+1\neq 0 \rightarrow x\neq -1$

$x\neq 0$

$x-1\neq 0 \rightarrow x\neq 1$

$x+2\neq 0 \rightarrow x\neq -2$

$\dfrac{3}{x-1}+\dfrac{1}{x+2}\neq 0 \rightarrow \dfrac{3x+6+x-1}{(x-1)(x+2)}=\dfrac{4x+5}{(x-1)(x+2)}\neq 0$

$4x+5\neq 0$

$x\neq \frac{-5}{4}$

$\boxed{D_f: \mathbb{R}-\{-1,0,1,-2,\frac{-5}{4}\}}$

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الف) $f(x)=\sqrt{\left(\left(\frac{1}{3}\right)^x-9\right)\left(32-4^x\right)} \rightarrow (3^{-x}-9)(32-2^{2x})\geq 0$

ریشه: $x=-2$ ، $x=\frac{5}{2}$

x		-2		$\frac{5}{2}$	
	$+$		$-$		$+$

$\boxed{D_f: (-\infty,-2]\cup[\frac{5}{2},+\infty)}$

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ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

(۱) $x-1\geq 0 \rightarrow x\geq 1$

(۲) $\sqrt{y+1}=3-\sqrt[4]{x-1} \rightarrow 3-\sqrt[4]{x-1}\geq 0 \rightarrow \sqrt[4]{x-1}\leq 3 \xrightarrow{(\ )^4} x-1\leq 81$

$x\leq 82$

$\boxed{D_f: [1,82]}$

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$f(x)=\dfrac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1}$ ⇝

(۱) $x^2-x-2>0 \rightarrow (x-2)(x+1)>0$

x		-1		2	
	$+$		$-$		$+$

$x: (-\infty,-1)\cup(2,+\infty)$

(۲) $x^2-1\geq 0 \rightarrow x^2\geq 1 \rightsquigarrow x\geq 1 \cup x\leq -1$

$\Rightarrow \boxed{D_f: (-\infty,-1)\cup(2,+\infty)}$

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$f(x)=\sqrt{3+ax-x^2} \rightarrow -x^2+ax+3\geq 0$

$D_f: [-2,b]$

x		-2		b	
	$-$		$+$		$-$

$f(-2)=0 \rightarrow -4-2a+3=0$, $2a=-1$, $\boxed{a=\frac{-1}{2}}$

$f(b)=0 \rightarrow -b^2+ab+3=0$

$-b^2-\frac{1}{2}b+3=0$

$b^2+\frac{1}{2}b-3=0$

$2b^2+b-6=0$

$b^2+b-12=0$

$(b+\frac{4}{2})(2b-3)=12 \rightarrow (b+2)(2b-3)=0$ ⟨ $b=-2$ ✗ ، $\boxed{b=\frac{3}{2}}$

$a+b=\frac{-1}{2}+\frac{3}{2}=1$

$P=ax^2+bx+c$ ⇝

x	
P	موافق a

x		x_1	
P	مخالف a		موافق a

x		x_1		x_2	
P	موافق		مخالف		موافق
	$-$		$+$		$-$

D_f

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$f(x)\begin{cases} 3x-2 & x\geq 1 \\ 2x+3 & x<1 \end{cases}$

$g(x)=\sqrt{f(x)-x} \rightarrow f(x)-x\geq 0 \rightarrow f(x)\geq x$

$\begin{cases} 3x-2\geq x \quad x\geq 1 \rightarrow 2x-2\geq 0 \quad x\geq 1 \\ \quad x\geq 1 \\ \cup \\ 2x+3\geq x \quad x<1 \rightarrow x+3\geq 0 \quad x<1 \\ \quad x\geq -3 \end{cases}$

$x: [-3,1)$

$\boxed{D_f: [-3,+\infty)}$

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$$f(x)\begin{cases}(a+1)(x+2) & x>1\\ 3a+2x & x\le 1\end{cases}$$

$$2f(5)=f(-2)+a \rightsquigarrow \begin{matrix} f(5)=(a+1)7 \\ f(-2)=3a-4\end{matrix} \Rightarrow \begin{matrix} 2\times 7(a+1)=3a-4+a \\ 14a+14=4a-4 \\ 10a=-18 \\ \boxed{a=-1/8}\end{matrix}$$ ۶

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2$$

$$2-\sqrt{3}=\left(\sqrt{\tfrac{1}{2}}-\sqrt{\tfrac{3}{2}}\right)^2$$

$$2+\sqrt{3}=\left(\sqrt{\tfrac{1}{2}}+\sqrt{\tfrac{3}{2}}\right)^2$$

$$f(2-\sqrt{3})+f(2+\sqrt{3})=\sqrt{2-\sqrt{3}}+\frac{1}{\sqrt{2-\sqrt{3}}}+2+\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}+2=\sqrt{\tfrac{3}{2}}-\cancel{\sqrt{\tfrac{1}{2}}}+\frac{1}{\sqrt{\tfrac{3}{2}}-\sqrt{\tfrac{1}{2}}}+\sqrt{\tfrac{3}{2}}+\cancel{\sqrt{\tfrac{1}{2}}}+\frac{1}{\sqrt{\tfrac{3}{2}}+\sqrt{\tfrac{1}{2}}}+4$$

$$\left|\sqrt{\tfrac{1}{2}}-\sqrt{\tfrac{3}{2}}\right|=\sqrt{\tfrac{3}{2}}-\sqrt{\tfrac{1}{2}} \qquad \sqrt{\tfrac{1}{2}}+\sqrt{\tfrac{3}{2}} \qquad \frac{\sqrt{3}-1}{\sqrt{2}} \qquad \frac{\sqrt{3}+1}{\sqrt{2}}$$

$$=2\sqrt{\tfrac{3}{2}}+\underbrace{\frac{\sqrt{2}}{\sqrt{3}-1}+\frac{\sqrt{2}}{\sqrt{3}+1}}_{\text{مخرج مشترک}}+4=2\sqrt{\tfrac{3}{2}}+\frac{\overset{2\sqrt{6}}{\sqrt{6}+\cancel{\sqrt{2}}+\sqrt{6}-\cancel{\sqrt{2}}}}{2}+4=\frac{\overset{2}{\cancel{2}}\sqrt{3}}{\sqrt{2}}+\sqrt{6}+4=\boxed{2\sqrt{6}+4}$$

$$\left(2f(x)-3f(-x)=4x^2-x\right)\times 2$$
$$\left(-3f(x)+2f(-x)=4x^2+x\right)\times 3$$

$$\rightsquigarrow \begin{matrix} 4f(x)-\cancel{6f(-x)}=8x^2-2x \\ -9f(x)+\cancel{6f(-x)}=12x^2+3x \\ \hline -5f(x)=20x^2+x \\ \boxed{f(x)=-4x^2-0/2x}\end{matrix}$$ ۸

$$(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1$$

$$x=0 \rightsquigarrow 2f(0)=3m-1 \rightarrow f(0)=\frac{3m-1}{2}$$

$$x=-2 \rightsquigarrow 6f(0)=16+2m+3m-1=15+5m \rightarrow f(0)=\frac{5m+15}{6}$$

$$\begin{cases}\dfrac{3m-1}{2}=\dfrac{5m+15}{6}\\ 10m+30=18m-6\\ 8m=36\\ m=\dfrac{36}{8}=\dfrac{9}{2}\end{cases}$$ ۹

$$f(0)=\frac{\frac{27}{2}-1}{2}=\boxed{\frac{25}{4}}$$

$$f(x)+f\left(\tfrac{1}{x}\right)=\frac{3x^2-12x+3}{x} \rightsquigarrow ax+b+\frac{a}{x}+b=ax+\frac{a}{x}+2b=3x+\frac{3}{x}-12$$

$$f(x)=ax+b$$

$$\begin{matrix}a=3\\ b=-6\end{matrix} \rightarrow y=3x-6$$ ۱۰

$$f(-1) \Rightarrow y=-3-6=-9$$