به نام خداوند دانش و ... یازدهم ریاضی گروه (

۲۰

Subject: (شماره تکلیف :: ۴)

Date: دبیرستان ... ۲

(۱) ۵

الف) $f(x) = \sqrt{\dfrac{x-1}{x} - \dfrac{x}{x-1}} = \sqrt{\dfrac{(x-1)(x-1)-[x(x)]}{x(x-1)}}$

$= \sqrt{\dfrac{x^2-2x+1-x^2}{x^2-x}} = \sqrt{\dfrac{-2x+1}{x(x-1)}}$ $\boxed{\text{زیر رادیکال} \geq ۰}$

$-2x+1=0 \Rightarrow -2x=-1$

$x = \dfrac{-1}{-2} = \dfrac{1}{2}$

$x(x-1)$: 0, 1

	0		$\frac{1}{2}$		1	
$+$	$\emptyset$	$-$	0	$+$	$\emptyset$	$-$

$D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \dfrac{\dfrac{1}{x+1} - \dfrac{2}{x}}{\dfrac{3}{x-1} + \dfrac{1}{x+2}}$ $\boxed{x \neq -1}$ $\boxed{x \neq 0}$ $\boxed{x \neq 1}$ $\boxed{x \neq -2}$ مخرج $\neq ۰$

$\dfrac{3}{x-1} + \dfrac{1}{x+2} \neq 0$

$\dfrac{\overbrace{3(x+2)}^{3x+6} + \overbrace{1(x-1)}^{x-1}}{(x-1)(x+2)} = \dfrac{4x+5}{x^2+x-2}$

$\Rightarrow 4x+5 \neq 0 \Rightarrow 4x \neq -5 \Rightarrow \boxed{x \neq \frac{-5}{4}}$

$D_f = \mathbb{R} - \{-1, 0, 1, -2, -\frac{5}{4}\}$

(۲) ۵

الف) $f(x) = \sqrt{\left(\left(\frac{1}{3}\right)^x - 9\right)\left(32 - 4^x\right)} \rightarrow 32 = 4^x$

$2^5 = 2^{2x} \Rightarrow 2x = 5 \Rightarrow \boxed{x = \frac{5}{2}}$

$\boxed{\text{زیر رادیکال} \geq ۰}$ $\boxed{\text{مخرج} \neq ۰}$

$\left(\frac{1}{3}\right)^x = 9$

$3^{-1(x)} = 9 = 3^2 \Rightarrow -x = 2 \Rightarrow \boxed{x = -2}$

	-2		$\frac{5}{2}$	
$+$	0	$-$	0	$+$

$D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

ب) $f(x) = \underbrace{\sqrt[4]{x-1}}_{\geq 0} + \underbrace{\sqrt{y+1}}_{\geq 0} = 3$

$x-1 \geq 0$

$x \geq 1$

$3 \geq \sqrt[4]{x-1} \geq 0$

$81 \geq x-1 \geq 0$

$82 \geq x \geq 1$

$D_f = [1, 82]$

Arman

۳ د

$$f(x)=\frac{\log_{4}(x^2-x-2)}{\sqrt{x^2-1}+1}$$

$x^2-x-2>0 \quad (x-2)(x+1)>0$; 2 , -1

$-1 \quad 2$: $+ \circ - \circ +$

$x^2-1\geq 0 \quad (x+1)(x-1)\geq 0$

$-1 \quad 1$: $+ \bullet - \bullet +$

$\underbrace{\sqrt{x^2-1}}_{\geq 0}+1\neq 0$ همواره برقرار است

$$D_f=(-\infty,-1)\cup(2,+\infty)$$

۴ د

$\sqrt{-x^2+ax+3}$ $\qquad D_f=[-2,b]$ $\qquad a+b$

$-2 \quad \frac{3}{2}$: $- \bullet + | -$

$=\sqrt{-(x^2-ax-3)}$

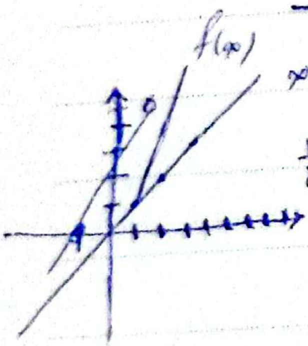
$-(x+2)(x+c)=x^2-ax-3$

$\Rightarrow -x^2+ax+3 \xrightarrow{x=-2} -4-2a+3=0$

$-2a-1=0 \Rightarrow -2a=1$

$\boxed{a=-\frac{1}{2}}$

$-(x^2+\frac{1}{2}x-3)=-x^2-\frac{1}{2}x+3$



$\frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow \frac{\frac{1}{2}\pm\sqrt{\frac{1}{4}-4\times-1\times3}}{-2}$ $\quad +12$

$\frac{1}{4}+\frac{12}{1}\,\frac{48}{4}=\frac{49}{4}$

$\sqrt{\Delta}=\frac{7}{2}$

$\frac{\frac{1}{2}+\frac{7}{2}}{-2}=2 \qquad \frac{\frac{1}{2}-\frac{7}{2}}{-2}=\frac{-3}{-2}=\frac{3}{2}$ $\quad \frac{-6}{2}$

$\boxed{-\frac{1}{2}+\frac{3}{2}=1}$

۵ د

$g(x)=\sqrt{f(x)-x}$ دامنه تابع

$$f(x)=\begin{cases}3x-2 & x\geq 1\\ 2x+3 & x<1\end{cases}$$

$f(x)-x\geq 0 \Rightarrow f(x)\geq x$

$D_f=[-3,+\infty)$

$\boxed{3x-2\geq x}\ x\geq 1$

$\Rightarrow 2x-2\geq 0 \quad 2x\geq 2 \Rightarrow x\geq 1$ ✓

$\boxed{2x+3\geq x}\ x<1 \qquad \cap\ -3\leq x<1$

$\Rightarrow 2x+3\geq x \Rightarrow x+3\geq 0 \Rightarrow x\geq -3$

(۴) $f(x) = \begin{cases} (a+1)(x+2) & x \geq 1 \\ 3a + 2x & x < 1 \end{cases}$ $\quad 2f(5) = f(-2) + a$ «مقدار a ؟؟؟؟»

$$2[(a+1)(5+2)] = 3a + 2(-2) + a$$

$$\underbrace{14a + 14}_{} = \underbrace{3a - 4 + a}_{4a-4} \Rightarrow 10a = -18$$

$$\boxed{a = -1.8}$$

(۷) $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$ $\quad f(\overbrace{2-\sqrt{3}}^{a}) + f(\overbrace{2+\sqrt{3}}^{\frac{1}{a}})$

$$a = 2 - \sqrt{3} \quad \frac{1}{a} = \frac{1}{2-\sqrt{3}} \cdot \frac{(2+\sqrt{3})}{(2+\sqrt{3})} = 2 + \sqrt{3}$$

$$\sqrt{a} + \frac{1}{\sqrt{a}} + 2 + \underbrace{\sqrt{\frac{1}{a}}}_{\frac{1}{\sqrt{a}}} + \underbrace{\frac{1}{\frac{1}{\sqrt{a}}}}_{\sqrt{a}} + 2$$

$$= 2\sqrt{a} + 2\frac{1}{\sqrt{a}} + 4$$

$$2 \times \sqrt{2-\sqrt{3}} + 2\frac{1(\sqrt{2+\sqrt{3}})}{\sqrt{2-\sqrt{3}}(\sqrt{2+\sqrt{3}})} + 4 = 2\underbrace{\sqrt{2-\sqrt{3}}}_{\sqrt{8-4\sqrt{3}}} + 2\underbrace{\sqrt{2+\sqrt{3}}}_{\sqrt{8+4\sqrt{3}}} + 4$$

$$A = \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}$$

$$A^2 = 2 - \sqrt{3} + 2 + \sqrt{3} + 2\sqrt{1} = 6 \Rightarrow A = \sqrt{6}$$

$$\boxed{2\sqrt{6} + 4}$$

(۸) $(2f(x) - 3f(-x) = 4x^2 - x) \times 2 \Rightarrow 4f(x) - 6f(-x) = 8x^2 - 2x$

$x \to -x \quad (-3f(x) + 2f(-x) = 4(-x)^2 + x) \times 3 \quad -9f(x) + 6f(-x) = 12x^2 + 3x$

$$\oplus \quad -5f(x) = 20x^2 + x$$

$$\Rightarrow f(x) = \frac{20x^2 + x}{-5} = -4x^2 - \frac{1}{5}x$$

$f(0)$ مقدار

(۹)

$$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$$

Subject : (   )   به نام خداوند دانش   Date :

$x \Rightarrow 0 \quad (0+2)\,f(0) - \underbrace{2\times 0\times f(0+2)}_{0} = 4\times 0^2 - m\times 0 + 3m - 1$

$(2f(0) = 3m-1)\times 3 \Rightarrow 6f(0) = 9m-3$

$x \Rightarrow -2 \quad \underbrace{(\cancel{-2+2})\,f(-2)}_{0} - 2\times -2\times f(0) = 4\times(-2)^2 - m\times(-2) + 3m - 1$

$6f(0) = 4\times 4 + 2m + 3m - 1 = \underbrace{\cancel{16-1}}_{15} + 5m$

$9m - 3 = 5m + 15$

$4m = 18 \Rightarrow \boxed{m = \frac{18}{4}} \Rightarrow 2f(0) = 3\times\frac{18}{4} - 1$

$\frac{54}{4} - \frac{4}{4} = \frac{50}{4}$

$f(0) = \frac{\frac{50}{4}}{\frac{2}{1}} = \boxed{\frac{25}{4}} = 6\frac{1}{4}$

(10)

$f(x) = ax + b$

$f(-1) = ????$

$f(\frac{1}{x}) = \frac{a}{x} + b$

$f(x) + f(\frac{1}{x}) = \frac{3x^2 - 12x + 3}{x}$

$\Rightarrow \frac{ax^2}{x} + \frac{bx}{x} + \frac{a}{x} + \frac{bx}{x} = \frac{ax^2 + 2bx + a}{x}$

$ax^2 + 2bx + a = 3x^2 - 12x + 3 \Rightarrow a = 3$

$2b = -12 \Rightarrow b = -6$

$f(x) = 3x - 6$

$\Rightarrow f(-1) = -3 - 6 = \boxed{-9}$

$-1\times 3 - 6$

— Arman