Subject: تکلیف شماره ۳ 1
Date

بنام خدا

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(1)

الف) $f(x) = \sqrt{\dfrac{x-1}{x} - \dfrac{x}{x-1}}$

$\rightarrow \dfrac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \dfrac{x^2+1-2x-x^2}{x^2-x} \geq 0 \rightarrow \dfrac{1-2x}{x(x-1)} \geq 0$

	0		$\frac{1}{2}$		1	
$+$		$-$	0	$+$		$-$

$\rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \dfrac{\dfrac{1}{x+1} - \dfrac{2}{x}}{\dfrac{3}{x-1} + \dfrac{1}{x+2}} \rightarrow$ اول مخرج های هر عبارت را بررسی کنید پس -1 و 0 و 1 و -2 تعریف نشده

$\xrightarrow[\text{مخرج}]{\text{ساده سازی}} \dfrac{3x+6+x-1}{(x-1)(x+2)} \neq 0 \rightarrow \dfrac{4x+5}{(x-1)(x+2)} \neq 0 \rightarrow 4x \neq -5 \rightarrow x \neq \dfrac{-5}{4}$

$\Rightarrow D_f = \mathbb{R} - \{-1, -2, 0, 1, \frac{-5}{4}\}$

(2)

$f(x) = \sqrt{\left(\left(\frac{1}{3}\right)^x - 9\right)\left(32 - 4^x\right)} \rightarrow$ عبارت ≥ 0

ریشه ها $\rightarrow \left(\frac{1}{3}\right)^x - 9 = 0 \rightarrow \left(\frac{1}{3}\right)^x = 9 \rightarrow x = -2$

$32 - 4^x = 0 \rightarrow 32 = 4^x \rightarrow 2^5 = 2^{2x} \rightarrow x = \frac{5}{2}$

$\rightarrow$

	-2		$\frac{5}{2}$	
$+$	0	$-$	0	$+$

$D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

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ب) $\sqrt[4]{x-1} + \sqrt{y+1} = 3 \to \sqrt{y+1} = 3 - \sqrt[4]{x-1}$

مثبت یا صفر

$\Rightarrow$ ① $3 - \sqrt{x-1} \geq 0 \Rightarrow 3 \geq \sqrt{x-1} \to 9 \geq x-1$

$\to 10 \geq x$ ② $x-1 \geq 0 \to x \geq 1 \Rightarrow D_f = [1, 10]$

③

$f(x) = \dfrac{\log(x^2-x-2)}{\sqrt{x^2-1}+1} \to x^2-x-2 > 0 \;\; (x-2)(x+1) > 0$

$\begin{array}{c|c|c} & -1 & 2 & \\ + & - & + \end{array}$ ①

$\sqrt{x^2-1} \geq 0 \to x^2-1 \geq 0 \to x^2 \geq 1 \Rightarrow x \geq 1 \,,\, x \leq -1$ ②

$\sqrt{x^2-1}+1 \neq 0$ ——— همواره برقرار ① ∩ ② $\to$ $-1 \quad 1 \quad 2$

$\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

④

$\sqrt{3+ax-x^2} \rightsquigarrow [-2, b] \qquad a+b = \frac{3}{2} - \frac{1}{2} = 1$ جواب

$3+ax-x^2 \geq 0 \xrightarrow{-2=x} \Rightarrow 3-2a-4 = 0 \to -1 = 2a$

$a = -\frac{1}{2}$

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$3 - \frac{1}{2}x - x^2 \geq 0 \xrightarrow{\times 2} 6 - x - 2x^2 \geq 0 \xrightarrow{\times -1} 2x^2 + x - 6 \leq 0$

$\xrightarrow{\text{ریشه}} x^2+x-12 \leq 0 \to (x+4)(x-3)$

$\begin{array}{c|c|c} & -2 & \frac{3}{2} \to b & \\ + & - & + \end{array}$

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$$f(x)=\begin{cases} \sqrt{2x-2} & ; x\geq 1 \\ \sqrt{x+3} & ; x<1 \end{cases}$$

① شرط $x\geq 1 \longrightarrow \sqrt{2x-2} \longrightarrow 2x-2\geq 0 \longrightarrow \underline{x\geq 1}$

② شرط $x<1 \longrightarrow \sqrt{x+3} \longrightarrow x+3\geq 0 \longrightarrow x\geq -3 \longrightarrow -3\leq x<1$

$\Rightarrow D_f=[-3,1)\cup[1,+\infty) \Rightarrow D_f=[-3,+\infty)$

(۶)

$$f(x)=\begin{cases} (a+1)(x+2) & ; x>1 \\ 3a+2x & ; x\leq 1 \end{cases}$$

$2f(5)=f(-2)+a$

جایگذاری $\longrightarrow 2\underbrace{(a+1)(x+2)}_{x=5}=\underbrace{3a+2x}_{x=-2}+a \Rightarrow 2(a+1)(7)=4a-4$

$\rightarrow 14a+14=4a-4 \rightarrow 10a=-18 \rightarrow \underline{a=-1.8}$

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$f(x)=\sqrt{x}+\dfrac{1}{\sqrt{x}}+2$

$f(2-\sqrt{3})+(f(2+\sqrt{3})$ شرط عبارت معکوس هم

$2-\sqrt{3}\times\dfrac{2+\sqrt{3}}{2+\sqrt{3}}=\dfrac{1}{2+\sqrt{3}}$

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ادامه ← پس فقط یکبار عبارت را حساب کرده به ازای یکی و ضرب در ۲ می کنیم چون در این تابع عدد با معکوسش جمع می شود و هر یک معکوس دیگری است!

$$f(2+\sqrt{3}) \longrightarrow f(x) = \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2$$

$$\rightarrow \sqrt{\frac{2\times(2+\sqrt{3})}{2}} + \sqrt{\frac{1}{\frac{2(2+\sqrt{3})}{2}}} + 2$$

$$4+2\sqrt{3} = \sqrt{\frac{(1+\sqrt{3})^2}{2}} \Rightarrow \frac{1+\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{2}}{1+\sqrt{3}} + 2 = \frac{4+2\sqrt{3}+2+2\sqrt{2}+2\sqrt{6}}{\sqrt{2}+\sqrt{6}}$$

$$= \frac{6+2\sqrt{3}+2\sqrt{2}+2\sqrt{6}}{\sqrt{2}+\sqrt{6}} = \frac{2\sqrt{3}(\sqrt{3}+1)+2\sqrt{2}(1+\sqrt{3})}{\sqrt{2}+\sqrt{6}}$$

$$\frac{\cancel{(1+\sqrt{3})}(2\sqrt{3}+2\sqrt{2})}{\sqrt{2}\,\cancel{(\sqrt{3}+1)}} = (\sqrt{6}+2)\times 2 = 2\sqrt{6}+4$$

جواب

(۸)

$$\begin{cases} (2f(x) - 3f(-x) = 4x^2 - x) \times 2 \\ (2f(-x) - 3f(x) = +4x^2 + x) \times 3 \end{cases}$$

$$\downarrow$$

$$\begin{cases} 4f(x) - \cancel{6f(-x)} = 8x^2 - 2x \\ \cancel{6f(-x)} - 9f(x) = 12x^2 + 3x \end{cases}$$

$$-5f(x) = 20x^2 + x \longrightarrow f(x) = -4x^2 - \frac{x}{5}$$

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$$f(0) \rightarrow (x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$$

$$\xrightarrow{x=-2} 6f(0) = 16 + 2m + 3m - 1 \rightarrow 6f(0) = 15 + 5m$$

$$\xrightarrow{x=0} 2f(0) = 3m - 1 \xrightarrow{\times 3} = 9m - 3 \Rightarrow$$

$$9m - 3 = 15 + 5m \rightarrow 4m = 18 \rightarrow m = 4.5$$

$$f(0) = \frac{3m-1}{2} \Rightarrow \frac{13.5-1}{2} = \frac{12.5}{2} = 6.25$$

10

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x}$$

$$\rightarrow ax + b + \frac{a}{x} + b = \frac{3x^2 - 12x + 3}{x} \xrightarrow{\times x} ax^2 + bx + a + bx$$

$$= 3x^2 - 12x + 3 \Rightarrow ax^2 + 2bx + a = 3x^2 - 12x + 3$$

$$\rightarrow a = 3, \quad b = -6 \Rightarrow f(x) = 3x - 6 \xrightarrow{f(-1)} -3 - 6 = -9$$