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الف) $\sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} \leq \sqrt{\frac{(x-1-x)(x-1+x)}{x(x-1)}} = \sqrt{\frac{-2x+1}{x(x-1)}}$

$$\begin{array}{c|c|c} 0 & \frac{1}{2} & 1 \\ \hline + & - & + \\ \hline \end{array}$$

$D_f \leq (-\infty, 0) \cup [\frac{1}{2}, 1)$

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ب) $\frac{x-2x-2}{(x+1)(x)} \leq \frac{x-2}{(x+1)(x)}$

$\frac{-x-2}{(x+1)(x)} \rightarrow \neq 0 \Rightarrow x \neq \{-1, 0\}$
 $\frac{3x+5}{(x-1)(x+2)} \rightarrow \neq 0 \Rightarrow x \neq \{-\frac{5}{3}\}$
 $\rightarrow \neq 0 \Rightarrow x \neq \{1, -2\}$

$\Rightarrow D_f \leq \mathbb{R} - \left\{ -1, 0, -\frac{5}{3}, 1, -2 \right\}$

الف) $\sqrt{((\frac{1}{x})^2 - 9)(2x - 2^2x)} \rightarrow (\frac{1}{x})^2 = 9 \Rightarrow x = -\frac{1}{3}$
 $\rightarrow 2x \leq 2 \Rightarrow 2x \leq 2 \rightarrow x \leq \frac{2}{2}$

$$\begin{array}{c|c|c} -2 & \frac{2}{2} & \\ \hline + & - & + \\ \hline \end{array}$$

$\Rightarrow D_f \leq (-\infty, -2] \cup [\frac{2}{2}, +\infty)$

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ب) $\sqrt{y-1} \leq \sqrt{x-1} \Rightarrow \sqrt{x-1} \leq 2 \rightarrow |x-1| \leq 1 \Rightarrow -1 \leq x-1 \leq 1$

$-10 \leq x \leq 12 \quad (D_1)$

$x-1 \geq 0 \rightarrow x \geq 1 \quad (D_2)$

$D_f \leq D_1 \cap D_2 \leq [1, 12]$

$2x-2-2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow \begin{array}{c|c|c} -1 & 2 & \\ \hline + & - & + \\ \hline \end{array} \quad D_3$

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$$\begin{aligned}
 x^2 - x - 2 > 0 &\rightarrow (x-2)(x+1) > 0 \rightarrow \begin{array}{c} -1 \quad 2 \\ + \quad | \quad - \quad | \quad + \end{array} D_1 \\
 x^2 - 1 \geq 0 &\rightarrow (x-1)(x+1) \geq 0 \rightarrow \begin{array}{c} -1 \quad 1 \\ + \quad | \quad - \quad | \quad + \end{array} D_2 \\
 \sqrt{x^2 - 1} \neq -1 &\rightarrow \text{crucial}
 \end{aligned}
 \left. \vphantom{\begin{aligned} x^2 - x - 2 > 0 \\ x^2 - 1 \geq 0 \end{aligned}} \right\} D_1 \cap D_2 = (-\infty, -1] \cup [2, +\infty)$$

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$$\begin{aligned}
 x = -2 &\rightarrow 2 - 2a - 4 \leq 0 \Rightarrow -2a \leq 1 \rightarrow a \in [-1/2, \infty) \\
 \Rightarrow \sqrt{-(x^2 + 1/4 x - 2)} &\rightarrow x \in \frac{-1/4 \pm \sqrt{1/16 + 8}}{2} \Rightarrow x_1 = -2, x_2 = -1/2 \\
 \Rightarrow D_f \in [-2, -1/2] = [-2, b] &\Rightarrow b = -1/2 \\
 \Rightarrow a + b = -2/4 = -1/2 &\Rightarrow a + b = 1
 \end{aligned}$$

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$$\begin{aligned}
 x \geq 1 &\rightarrow f(x) \leq 2x - 2 \Rightarrow g(x) \leq \sqrt{2x - 2} = \sqrt{2(x-1)} \rightarrow 2(x-1) \geq 0 \Rightarrow x \geq 1 \quad (D_1) \\
 x < 1 &\rightarrow f(x) \leq 2x + 2 \Rightarrow g(x) \leq \sqrt{2x + 2} = \sqrt{2(x+1)} \rightarrow 2(x+1) \geq 0 \xrightarrow{x < 1} -1 \leq x < 1 \quad (D_2) \\
 D_g &\subset D_1 \cup D_2 = [-1, +\infty)
 \end{aligned}$$

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$$\begin{aligned}
 f(a) \leq f(-2) + a &\rightarrow \sqrt{(a+1)(a+2)} = 2a - 4 + a \rightarrow 1/4 a + 1/4 \leq 2a - 4 + a \\
 10a &\leq -1/4 \Rightarrow a \leq -1/40
 \end{aligned}$$

$$(x - \sqrt{x})(x + \sqrt{x}) \leq 1 \rightarrow \text{عبر ضرب} \Rightarrow \sqrt{x - \sqrt{x}} + \frac{1}{\sqrt{x - \sqrt{x}}} + \sqrt{x + \sqrt{x}} + \frac{1}{\sqrt{x + \sqrt{x}}} + 4$$

$$= x(\underbrace{\sqrt{x + \sqrt{x}} + \sqrt{x - \sqrt{x}}}_A) + 4$$

$$A^2 = x + \sqrt{x} + x - \sqrt{x} + 2\sqrt{1} = 4 \Rightarrow A = \sqrt{4}$$

$$\Rightarrow x(\sqrt{4}) + 4 = 2\sqrt{4} + 4$$

$$\begin{cases} xf(x) - x^2 f(-x) = 6x^2 - x \\ xf(-x) - x^2 f(x) = 6x^2 + x \end{cases}$$

$$\rightarrow \begin{cases} xf(x) - 4f(-x) = 14x^2 - 2x \\ 4f(-x) - 9f(x) = 14x^2 + 3x \\ \hline -5f(x) = 20x^2 + x \\ \Rightarrow f(x) = -4x^2 - \frac{x}{5} \end{cases}$$

$$xf(0) = x^{m-1} \quad (x \leq 0) \rightarrow f(0) = \frac{x^{m-1}}{x}$$

$$4f(0) = 14 + x^m + x^{m-1} = 14 + 4m \quad (x = -1)$$

$$4\left(\frac{x^{m-1}}{x}\right) = 14 + 4m \rightarrow 4m - 3 = 14 + 4m \rightarrow 4m = 17 \Rightarrow m = \frac{17}{4} = 4\frac{1}{4}$$

$$\Rightarrow f(0) = \frac{x^{(\frac{17}{4})-1}}{x} = \frac{17x^{\frac{13}{4}}}{4} = 9, 24 = \frac{24}{4}$$

$$ax + b + \frac{9}{x} + b = \frac{6x^2 - 12x + 9}{x}$$

$$x(ax + 2b + \frac{9}{x}) = 6x^2 - 12x + 9$$

$$ax^2 + 2bx + 9 = 6x^2 - 12x + 9 \Rightarrow a = 6, 2b = -12 \Rightarrow b = -6$$

$$\rightarrow f(x) = ax + b = 6x - 6 \rightarrow f(-1) = 6(-1) - 6 = -12$$