

الف)  $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1}$  ①

$$\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \quad \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \quad \frac{-2x+1}{x(x-1)} \geq 0$$

$$+ \quad | \quad - \quad | \quad + \quad | \quad -$$

$$Df = (-\infty, 0) \cup [\frac{1}{2}, 1)$$

ب)  $f(x) = \frac{\frac{1}{x+1} - \frac{x}{x}}{\frac{x}{x-1} + \frac{1}{x+1}} = \frac{\frac{x - x^2 - x^2}{x(x+1)}}{\frac{x^2 + x + x - 1}{x(x+1)}} = \frac{-x^2 - x}{x^2 + 2x - 1}$

$$Df = \mathbb{R} - \left\{ 0, -1, -\frac{2}{x}, -2, 1 \right\}$$

الف)  $f(x) = \sqrt{\left(\frac{1}{x}\right)^x - 9} (3x - x^2)$  ②

$$\left(\frac{1}{x}\right)^x - 9 = 0$$

$$3x = x^2 = x^0$$

$$\left(\frac{1}{x}\right)^x = 9$$

$$x = -2$$

$$x = \frac{2}{x}$$

$$-2 \quad \frac{2}{x}$$

$$Df = (-\infty, -2] \cup \left[\frac{2}{x}, +\infty\right)$$

ب)  $\sqrt[3]{x-1} + \sqrt{y+1} = 3$

(I)  $x-1 \geq 0 \rightarrow x \geq 1$

(II)  $\sqrt[3]{x-1} - 3 = -\sqrt{y+1}$   
 با توجه به  $\sqrt{y+1} \geq 0$

$$\sqrt[3]{x-1} - 3 \leq 0$$

$$\sqrt[3]{x-1} \leq 3 \rightarrow x-1 \leq 27$$

(I)  $\cap$  (II)  $\rightarrow Df = [1, 28]$

Scanned with

$$f(x) = \frac{\log(x^p - x - r)}{\sqrt{x^p - 1} + 1}$$

(μ)

$$x^p - x - r > 0 \rightarrow (x-r)(x+1) > 0 \quad \frac{-1 \quad r}{+ \quad - \quad - \quad +} \quad (I) \rightarrow (-\infty, -1) \cup (r, +\infty)$$

$$x^p - 1 > 0 \rightarrow x^p > 1 \rightarrow (-\infty, -1] \cup [1, +\infty)$$

$$\sqrt{x^p - 1} \neq -1 \rightarrow \text{ok}$$

$$Df = (-\infty, -1) \cup (r, +\infty)$$

$$\sqrt{r^p + ax - x^p} \quad [-r, b]$$

(κ)

$$r^p - ra - r = 0 \rightarrow -1 = ra \rightarrow \boxed{a = -\frac{1}{r}}$$

$$-x^p - \frac{1}{r}x + r \geq 0 \quad \Delta = b^p \quad \text{rac} = \frac{1}{r} - r(r)(-1) = \frac{r^2}{r}$$

$$\frac{+\frac{1}{r} \pm \sqrt{\frac{r^2}{r}}}{-r} \rightarrow \begin{cases} \frac{+\frac{1}{r} + \frac{r}{r}}{-r} = \frac{r}{-r} = \textcircled{-r} \\ \frac{+\frac{1}{r} - \frac{r}{r}}{-r} = \frac{-\frac{r^2}{r}}{-r} = \frac{-r}{-r} = \frac{r}{r} \end{cases}$$

$$\boxed{b = \frac{r}{r}}$$

$$a+b = -\frac{1}{r} + \frac{r}{r} = \textcircled{1}$$

$$f(x) = \begin{cases} rx - r & x \geq 1 \\ rx + r & x < 1 \end{cases} \quad g(x) = \sqrt{f(x) - x} \quad \textcircled{\lambda}$$

$$f(x) \geq x$$

$$(I) \rightarrow rx - r \geq x \rightarrow rx \geq r \rightarrow x \geq 1 \quad [1, +\infty)$$

$$(II) \rightarrow rx + r \geq x \rightarrow x \geq -r \quad [-r, 1) \rightarrow \text{بالنسبة لـ } r > 0$$

$$(I) \cup (II) \rightarrow Df = [-r, +\infty)$$

$$f(x) = \begin{cases} (a+1)(x+2) & x > 1 \\ 3a+2x & x < 1 \end{cases}$$

$$xf(a) = f(-2) + a$$

(4)

$$13a+12 = 3a-2+a$$

$$f(a) = (a+1)(2) = 2a+2$$

$$10a = -12 \rightarrow a = \frac{-12}{10} = -1.2$$

$$f(-2) = 3a-2$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$$

$$f(2-\sqrt{2}) + f(2+\sqrt{2}) = ?$$

(5)

$$(2-\sqrt{2})(2+\sqrt{2}) = 1 \rightarrow \text{ساده شود}$$

$$f(2-\sqrt{2}) = \sqrt{2-\sqrt{2}} + \frac{1}{\sqrt{2-\sqrt{2}}} + 2 = \sqrt{2-\sqrt{2}} + \sqrt{2+\sqrt{2}} + 2$$

$$f(2+\sqrt{2}) = \sqrt{2+\sqrt{2}} + \frac{1}{\sqrt{2+\sqrt{2}}} + 2 = \sqrt{2+\sqrt{2}} + \sqrt{2-\sqrt{2}} + 2$$

$$f(2-\sqrt{2}) + f(2+\sqrt{2}) = 2 + 2(\sqrt{2-\sqrt{2}} + \sqrt{2+\sqrt{2}}) = \boxed{4 + 2\sqrt{4}} \leftarrow \text{پایان}$$

$$\sqrt{2-\sqrt{2}} + \sqrt{2+\sqrt{2}} = A$$

A

$$A^2 = 2-\sqrt{2} + 2+\sqrt{2} + 2(\sqrt{2-\sqrt{2}} \times \sqrt{2+\sqrt{2}}) = 4 \rightarrow A = 2$$

$$f(x) = ?$$

$$xf(x) - xf(-x) = kx^2 - x$$

(6)

$$\begin{cases} xf(-x) - xf(x) = kx^2 + x \\ xf(x) - xf(-x) = kx^2 - x \end{cases}$$

$$3 \times \begin{cases} xf(-x) - xf(x) = kx^2 + x \\ -xf(-x) + xf(x) = kx^2 - x \end{cases}$$

$$\begin{cases} 4f(-x) - 4f(x) = 12x^2 + 3x \\ -4f(-x) + 4f(x) = 12x^2 - 3x \end{cases}$$

$$3 \times \begin{cases} -xf(-x) + xf(x) = kx^2 - x \end{cases}$$

$$\begin{cases} -4f(-x) + 4f(x) = 12x^2 - 3x \end{cases}$$

$$-2f(x) = kx^2 + x$$

$$f(x) = \frac{kx^2 + x}{-2}$$

Scanned with

$$(x+r)f(x) - rf(x+r) = rx^r - mx + rm - 1 \quad f(0) = ? \quad (9)$$

$$x \rightarrow (-r) \quad +4f(0) = 14 + rm + rm - 1 = 2m + 12$$

$$x \rightarrow 0 \quad rf(0) = rm - 1$$

$$r(rm - 1) = 2m + 12 \rightarrow 9m - r = 2m + 12$$

$$rm = 11 \rightarrow m = \frac{11}{r} = \frac{9}{r}$$

$$f(0) = \frac{r\left(\frac{9}{r}\right) - 1}{r} = \frac{\frac{9r}{r} - 1}{r} = \frac{\frac{9}{1} - 1}{\frac{1}{1}} = \frac{8}{1} = 8$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{rx^r - 1rx + r}{x} \quad f(-1) = ? \quad (10)$$

$$f(-1) + f(-1) = \frac{r + 1r + r}{-1} = -11$$

$$rf(-1) = -11$$

$$f(-1) = -9$$