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سارینا اسپناني ـ یازدهم دفتر C ـ شماره تکلیف ۴

①

الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}$

$$\frac{x-1}{x}-\frac{x}{x-1}\geq 0 \qquad \frac{x^2-2x+1-x^2}{x(x-1)}\geq 0 \qquad \frac{-2x+1}{x(x-1)}\geq 0$$

$\boxed{x=\frac{1}{2}}$, 0 , 1

	0		$\frac{1}{2}$		1	
$+$	ت	$-$	0	$+$	ت	$-$

$$D_f=(-\infty,0)\cup[\tfrac{1}{2},1)$$

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ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \rightarrow \dfrac{\frac{x-2x-2}{x^2+x}}{\frac{3x+6+x-1}{x^2+x-2}}=\dfrac{\frac{-x-2}{x(x+1)}}{\frac{4x+5}{(x+2)(x-1)}}$

$$D_f=\mathbb{R}-\{0,-1,-\tfrac{5}{4},-2,1\}$$

②

الف) $f(x)=\sqrt{\left[\left(\frac{1}{3}\right)^x-9\right](32-4^x)}$

$$\left(\frac{1}{3}\right)^x-9=0 \qquad\qquad 32=4^x=2^5$$

$$\left(\frac{1}{3}\right)^x=9 \rightarrow \boxed{x=-2} \qquad\qquad \boxed{x=\frac{5}{2}}$$

	-2		$\frac{5}{2}$	
$+$	0	$-$	0	$+$

$$D_f=(-\infty,-2]\cup[\tfrac{5}{2},+\infty)$$

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ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$(I)\rightarrow x-1\geq 0 \rightarrow x\geq 1$

$(II)\rightarrow \sqrt[4]{x-1}-3=\underbrace{-\sqrt{y+1}}_{\text{عبارت } \ominus \text{ یا صفر}}$ $\qquad \sqrt[4]{x-1}-3\leq 0$

$\sqrt[4]{x-1}\leq 3 \quad , \quad x-1\leq 81$

$\boxed{x\leq 82}$

$(I)\cap(II)\rightarrow D_f=[1,82]$

SUBO

③

$$f(x) = \frac{\log_4 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$$

$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0$ $\quad \begin{array}{ccccc} & -1 & & 2 & \\ + & \phi & - & \phi & + \end{array}$ $\quad (I) \rightarrow (-\infty, -1) \cup (2, +\infty)$

$x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow (-\infty, -1] \cup [1, +\infty)$

$\sqrt{x^2 - 1} \neq -1 \rightarrow$ عبارت همواره برقراره

$D_f = (-\infty, -1) \cup (2, +\infty)$

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④

$\sqrt{3 + ax - x^2} \qquad [-2, b]$

$3 - 2a - 4 = 0 \rightarrow -1 = 2a \rightarrow \boxed{a = \frac{-1}{2}}$

$-x^2 - \frac{1}{2}x + 3 \geq 0 \qquad \Delta = b^2 - 4ac = \frac{1}{4} - 4(3)(-1) = \frac{49}{4}$

$$\frac{+\frac{1}{2} \pm \sqrt{\frac{49}{4}}}{-2} \rightarrow \begin{cases} \dfrac{+\frac{1}{2} + \frac{7}{2}}{-2} = \dfrac{4}{-2} = -2 \\ \dfrac{+\frac{1}{2} - \frac{7}{2}}{-2} = \dfrac{-\frac{6}{2}}{-2} = \dfrac{-3}{-2} = \dfrac{3}{2} \end{cases}$$

$\boxed{b = \frac{3}{2}} \qquad a + b = \frac{-1}{2} + \frac{3}{2} = 1$

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⑤

$$f(x) = \begin{cases} 3x - 2 & x \geq 1 \\ 2x + 3 & x < 1 \end{cases} \qquad g(x) = \sqrt{f(x) - x}$$

$f(x) \geq x$

$(I) \rightarrow 3x - 2 \geq x \rightarrow 2x \geq 2 \rightarrow x \geq 1 \qquad [1, +\infty)$

$(II) \rightarrow 2x + 3 \geq x \rightarrow x \geq -3 \qquad [-3, 1) \rightarrow$ با توجه به محدودیت ضابطه

$(I) \cup (II) \rightarrow D_f = [-3, +\infty)$

$$f(x)=\begin{cases}(a+1)(x+2) & x>1\\ 3a+2x & x\leq 1\end{cases}$$

$2f(5)=f(-2)+a$ ⑥

$14a+14=3a-4+a$

$f(5)=(a+1)(7)=7a+7$

$10a=-18 \longrightarrow a=\frac{-18}{10}=-1/8$

$f(-2)=3a-4$

$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2$

$f(2-\sqrt{3})+f(2+\sqrt{3})=?$ ⑦

$(2-\sqrt{3})(2+\sqrt{3})=1$ ← پس وارون و معکوس هم اند

$f(2-\sqrt{3})=\sqrt{2-\sqrt{3}}+\frac{1}{\sqrt{2-\sqrt{3}}}+2=\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}+2$

$f(2+\sqrt{3})=\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}+2=\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}+2$

$f(2-\sqrt{3})+f(2+\sqrt{3})=4+2(\underbrace{\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}}_{A})=\boxed{4+2\sqrt{6}}$ ← پاسخ

$\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}=A$

$A^2=2-\cancel{\sqrt{3}}+2+\cancel{\sqrt{3}}+2(\sqrt{2-\sqrt{3}}\times\sqrt{2+\sqrt{3}})=6 \rightarrow A=\sqrt{6}$

$f(x)=?$ $\quad 2f(x)-3f(-x)=4x^2-x$ ⑧

$$\begin{cases}2f(-x)-3f(x)=4x^2+x\\ 2f(x)-3f(-x)=4x^2-x\end{cases}$$

$$\begin{array}{l}3\times\\ 2\times\end{array}\begin{cases}2f(-x)-3f(x)=4x^2+x\\ -3f(-x)+2f(x)=4x^2-x\end{cases}\qquad\begin{cases}\cancel{6f(-x)}-9f(x)=12x^2+3x\\ \cancel{-6f(-x)}+4f(x)=8x^2-2x\end{cases}$$

$-5f(x)=20x^2+x$

$$\boxed{f(x)=\frac{20x^2+x}{-5}}$$

$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1 \qquad f(0) = ?$ 9

$x \to -2 \qquad +6f(0) = 16 + 2m + 3m - 1 = 5m + 15$

$x \to 0 \qquad 2f(0) = 3m - 1$

$3\underbrace{(3m-1)}_{2f(0)} = 5m + 15 \longrightarrow 9m - 3 = 5m + 15$

$\boxed{4m = 18 \to m = \frac{18}{4} = \frac{9}{2}}$

$f(0) = \dfrac{3(\frac{9}{2}) - 1}{2} = \dfrac{\frac{27}{2} - 1}{2} = \dfrac{\frac{25}{2}}{\frac{2}{1}} = \dfrac{25}{4}$

$f(x) + f(\frac{1}{x}) = \dfrac{3x^2 - 12x + 3}{x} \qquad f(-1) = ?$ 10

$f(-1) + f(-1) = \dfrac{3 + 12 + 3}{-1} = -18 \qquad 2f(-1) = -18$

$\boxed{f(-1) = -9}$