آزمون ریاضی ۲۰

۱)

$$\lim_{x\to 1}\frac{4x^2-7x+3}{5x^2-8x+3}\quad \frac{0}{0}\ \text{رفع ابهام}\quad \frac{(x-1)(4x-3)}{(5x-3)(x-1)}=\frac{4x-3}{5x-3}=\frac{4-3}{5-3}=+\frac{1}{2}$$

۲

۲)

$$\lim_{x\to 0}\frac{|3x-1|-|3x+1|}{x}=\frac{1-3x-3x-1}{x}=\frac{-6x}{x}=-6$$

۲

۳)

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}=\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2}=\sqrt{x}+2=4$$

۲

۴)

$$\lim_{x\to 2}\frac{x-\sqrt{2x}}{2x^2-x-6}=\frac{\sqrt{x}(\sqrt{x}-\sqrt{2})}{(x-2)(2x+3)}=\frac{\sqrt{x}(\sqrt{x}-\sqrt{2})}{(\sqrt{x}-\sqrt{2})(\sqrt{x}+\sqrt{2})(2x+3)}=\frac{\sqrt{x}}{(\sqrt{x}+\sqrt{2})(2x+3)}$$

$$\frac{\sqrt{2}}{7(2\sqrt{2})}=\frac{1}{14}$$

۲

۵)

$$\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{5-x}}\times\frac{1+\sqrt{x}}{1+\sqrt{x}}\times\frac{2+\sqrt{5-x}}{2+\sqrt{5-x}}=\frac{1-x}{x-1}\times\frac{4}{2}=-2$$

۲

$\frac{0}{0}$ ← رفع ابهام

۶)

$$\lim_{x\to 4}\frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3}\times\frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4}\times\frac{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}=$$

$$\frac{27\times 3}{8\times 5}=\frac{81}{40}$$

۲

(7

$$\lim_{x\to 1} \frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1} \times \frac{\sqrt[3]{x^2}+1+\sqrt[3]{x}}{\sqrt[3]{x^2}+1+\sqrt[3]{x}} \times \frac{\sqrt{3x+\sqrt{x}}+2}{\sqrt{3x+\sqrt{x}}+2} =$$

$$\frac{3x+\sqrt{x}-4}{x-1} \times \frac{3}{4} = \frac{(3\sqrt{x}+4)(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = \frac{3\sqrt{x}+4}{\sqrt{x}+1} \times \frac{3}{4}$$

(2)

$$\lim \quad \frac{7}{2} \times \frac{3}{4} = \frac{21}{8}$$

(8

$$1+1+1$$

$$\lim_{x\to \pi} \frac{1+\cos^3 x}{\sin^2 x} = \frac{(1+\cos x)(1-\cos x+\cos^2 x)}{(1-\cos x)(1+\cos x)} = \frac{3}{+2}$$

(2)

$\frac{0}{0} \longrightarrow$ رفع ابهام

(9

$$\lim_{x\to \frac{\pi}{4}} \frac{1-\tan x}{\sin x-\cos x} = \frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x} = \frac{\frac{\cos x-\sin x}{\cos x}}{\sin x-\cos x} = \frac{-1}{\cos x} = \frac{-2}{\sqrt{2}}$$

(2)

$\frac{0}{0}$ رفع ابهام

$$\frac{-2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$$

(10

$$\lim_{x\to \frac{3\pi}{4}} \frac{\tan^2 x-1}{\cos 2x} = \frac{(\tan x+1)(\tan x-1)}{\cos^2 x-\sin^2 x} = \frac{\left(\frac{\sin x+\cos x}{\cos x}\right)\left(\frac{\sin x-\cos x}{\cos x}\right)}{(\cos x-\sin x)(\cos x+\sin x)}$$

$$= \frac{1}{\cos^2 x} = \frac{1}{\frac{1}{2}} = 2$$

(2)

$\frac{0}{0} \longrightarrow$ رفع ابهام

$$\tan^2 x+1 = \frac{1}{\cos x} - 2$$