نام/نام خانوادگی: آیلین عظیمی — تکلیف: ۲۰ — ۲۰ — کلاس: یکشنبه یازدهم دهم

$$\lim_{x\to 1}\frac{4x^2-7x+3}{5x^2-8x+3}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\lim_{x\to 1}\frac{4\cancel{(x-1)}(x-\frac{3}{4})}{5\cancel{(x-1)}(x-\frac{3}{5})}=\frac{4(1-\frac{3}{4})}{5(1-\frac{3}{5})}=\frac{1}{2}$$ ۱

۴

$$\lim_{x\to 0}\frac{|3x-1|-|3x+1|}{x}=\frac{0}{0}$$

$0^+:\ \dfrac{\overset{\ominus}{|3x-1|}-\overset{\oplus}{|3x+1|}}{x}=\dfrac{-3x+\cancel{1}-3x-\cancel{1}}{x}=\dfrac{-6x}{x}=-6$

$0^-:\ \dfrac{\overset{\ominus}{|3x-1|}-\overset{\oplus}{|3x+1|}}{x}=\dfrac{-3x+1-3x-1}{x}=-6$ ۲

۴

$$=-6$$

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\lim_{x\to 4}\frac{(\sqrt{x}+2)\cancel{(\sqrt{x}-2)}}{\cancel{(\sqrt{x}-2)}}=\sqrt{4}+2=4$$

۴

۳

$$\lim_{x\to 2}\frac{x-\sqrt{2x}}{2x^2-x-6}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\lim\frac{x-\sqrt{2x}}{2x^2-x-6}\times\frac{x+\sqrt{2x}}{x+\sqrt{2x}}=\frac{\overbrace{x^2-2x}^{x\cancel{(x-2)}}}{(\underbrace{2x^2-x-6}_{\cancel{(x-2)}(2x+3)})(x+\sqrt{2x})}$$

۴

$$=\lim_{x\to 2}\frac{x}{(2x+3)(x+\sqrt{2x})}=\frac{2}{7(4)}=\frac{1}{14}$$ ۴

$$\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{5-x}}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{5-x}}\times\frac{2+\sqrt{5-x}}{2+\sqrt{5-x}}=\frac{(1-\sqrt{x})(\overbrace{2+\sqrt{5-x}}^{2+\sqrt{4}=4})}{4-5+x}=\frac{(1-\sqrt{x})\times 4}{x-1}$$

۴ ۵

$$=\frac{4(\overset{-1}{\cancel{1-\sqrt{x}}})}{(\sqrt{x}+1)\cancel{(\sqrt{x}-1)}}=\lim_{x\to 1}\frac{-4}{\sqrt{x}+1}=\frac{-4}{2}=-2$$

$$\lim_{x\to 4}\frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\lim_{x\to 4}\frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3}\times\frac{(2)}{(2)}\times\frac{(1)}{(1)}=\frac{3x+4-16}{\underbrace{\sqrt{3x+4}+4}_{8}}\times\frac{\overbrace{\sqrt[3]{(5x+7)^2}+3\sqrt[3]{5x+7}+9}^{27}}{5x+7-27}$$ ۶

$$=\frac{27}{8}\times\frac{\overset{3(x-4)}{3x-12}}{\underset{5(x-4)}{5x-20}}=\lim\frac{27}{8}\times\frac{3}{5}=\frac{81}{40}$$

(۴)

$$\lim_{x\to 1}\frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\lim_{x\to 1}\frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1}\times\frac{(2)}{(2)}\times\frac{(1)}{(1)}=\frac{3x+\sqrt{x}-4}{\underbrace{\sqrt{3x+\sqrt{x}}+2}_{4}}\times\frac{\overbrace{\sqrt[3]{x^2}+\sqrt[3]{x}+1}^{3}}{x-1}$$ ۷

$$=\frac{3}{4}\times\frac{3x+\sqrt{x}-4}{x-1}=\frac{3}{4}\times\frac{3(\sqrt{x}-1)(\sqrt{x}+\frac{4}{3})}{(\sqrt{x}+1)(\sqrt{x}-1)}=\lim_{x\to 1}\frac{\frac{9}{4}(\sqrt{x}+\frac{4}{3})}{\sqrt{x}+1}=\frac{\frac{9}{4}\times\frac{7}{3}}{2}=\frac{21}{8}$$

(۴)

$$\lim_{x\to\pi}\frac{1+\cos^3 x}{\sin^2 x}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\lim_{x\to\pi}\frac{1+\cos^3 x}{\sin^2 x}=\frac{\cancel{(1+\cos x)}(1-\cos x+\cos^2 x)}{\underset{(\cancel{1+\cos x})(1-\cos x)}{(1-\cos^2 x)}}$$ ۸

(۱)

$$=\lim_{x\to\pi}\frac{1-\cos x+\cos^2 x}{1-\cos x}=\frac{1-(-1)+1}{1-(-1)}=\frac{3}{2}$$

$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x}=\frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x}=\frac{\overset{-1}{\cancel{\cos x-\sin x}}}{\cos x(\cancel{\sin x-\cos x})}$$ ۹

(۲)

$$=\lim_{x\to\frac{\pi}{4}}\frac{-1}{\cos x}=\frac{-1}{\frac{\sqrt{2}}{2}}=-\sqrt{2}$$

$$\lim_{x\to\frac{3\pi}{4}}\frac{\tan^2 x-1}{\cos 2x}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\lim_{x\to\frac{3\pi}{4}}\frac{\tan^2 x-1}{\cos 2x}=\frac{(\overbrace{\tan x}^{\frac{\sin x}{\cos x}}+1)(\tan x-1)}{\cos 2x}=\frac{\left(\frac{\sin x+\cos x}{\cos x}\right)\left(\frac{\sin x-\cos x}{\cos x}\right)}{\underset{\cos^2 x-\sin^2 x}{\cos 2x}}$$ ۱۰

(۴)

$$=\frac{\overset{-1}{\cancel{\sin^2 x-\cos^2 x}}}{\cos^2 x(\cancel{\cos^2 x-\sin^2 x})}=\lim_{x\to\frac{3\pi}{4}}\frac{-1}{\cos^2 x}=\frac{-1}{(\frac{\sqrt{2}}{2})^2}=\frac{-1}{\frac{1}{2}}=-2$$