

سوال (۱)

الف) $y = \frac{x+3}{2x^3+3x^2-11x+3}$

$$\begin{array}{r|l} 2x^3+3x^2-11x+3 & x-1 \\ -2x^3+2x^2 & \\ \hline 5x^2-11x+3 & \end{array}$$

$z = \frac{x+3}{(x-1)(2x^2+5x-3)}$
 $\downarrow$
 $\textcircled{1} \quad \textcircled{-3} \rightarrow \textcircled{\frac{1}{2}}$

$$\begin{array}{r} 5x^2-11x+3 \\ -5x^2+5x \\ \hline -6x+3 \\ +6x-3 \\ \hline 0 \end{array}$$

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$2x^2+5x-3 = 0 \rightarrow x^2+5x-3 = 0$

$\rightarrow (x+4)(x-1) = 0 \Rightarrow x = -4, x = 1$
 $\rightarrow x = -\frac{4}{2} = -2, x = \frac{1}{2}$
 $D_f = \mathbb{R} - \{1, -\frac{1}{2}\}$

ب) $y = \frac{x+3}{2x^3+9x^2+11x+3}$

$$\begin{array}{r|l} 2x^3+9x^2+11x+3 & x+1 \\ -2x^3-2x^2 & \\ \hline 11x^2+11x+3 & \end{array}$$

$z = \frac{x+3}{(x+1)(2x^2+7x+3)}$
 $\textcircled{-1} \quad \textcircled{-3} \rightarrow \textcircled{\frac{-1}{2}}$

$$\begin{array}{r} 11x^2+11x+3 \\ -11x^2-11x \\ \hline 3 \\ -3x-3 \\ \hline -3x-3 \\ +3x+3 \\ \hline 0 \end{array}$$

$2x^2+7x+3 = 0 \Rightarrow x = -1, x = -\frac{3}{2}$

$x = \frac{-7 \pm \sqrt{49-24}}{4} = \frac{-7 \pm 5}{4}$
 $\rightarrow x = -\frac{1}{2}, x = -3$
 $D_f = \mathbb{R} - \{-1, -\frac{3}{2}\}$

الف) $y = \frac{x+3}{x^3-2x^2+2x-1}$

$(x-1)(x^2-x+1) \neq 0 \Rightarrow x \neq 1$

$$\begin{array}{r|l} x^3-2x^2+2x-1 & x-1 \\ -x^3+2x^2 & \\ \hline 0x^2+2x-1 & \end{array}$$

$-x^3+2x-1$
 $+x^3-2x^2$
 $\hline -2x^2+2x-1$
 $+2x^2-2x$
 $\hline -1$
 $+x-1$
 $\hline x-1$

$D_f = \mathbb{R} - \{1\}$

$$y z \sqrt{\frac{n+3}{n^2-2n+1}} \rightarrow \frac{n+3}{n^2-2n+1} > 0 \Rightarrow \frac{n+3}{(n-1)(n^2-n+1)} > 0$$

$$D_f z (-\infty, -3] \cup (1, +\infty)$$

$$y \frac{2}{n^2-\omega|n-1|-2n+\omega} \rightarrow n^2-\omega|n-1|-2n+\omega \neq 0$$

$$n \geq 1 \rightarrow n^2-\omega n+\omega-2n+\omega \neq 0 \Rightarrow n^2-7n+2\omega \neq 0 \Rightarrow (n-2)(n-\omega) \neq 0$$

$$\Rightarrow n \neq 2, n \neq \omega$$

$$n < 1 \rightarrow n^2+\omega n-\omega-2n+\omega \neq 0 \Rightarrow n^2+3n \neq 0 \Rightarrow n(n+3) \neq 0 \Rightarrow n \neq 0, n \neq -3$$

$$D_f z \mathbb{R} - \{2, \omega, 0, -3\}$$

$$y z \frac{n+3}{|2n+1|-|n+3|} \rightarrow |2n+1|-|n+3| \neq 0$$

$$|2n+1| \neq |n+3| \rightarrow 2n+1 \neq n+3$$

$$D_f z \mathbb{R} - \{2, -\frac{4}{3}\}$$

$$\rightarrow 2n+1 \neq -n-3 \quad n \neq 2$$

$$\Rightarrow 3n \neq -4 \quad n \neq -\frac{4}{3}$$

$$y = \sqrt{|2n+1|-|n+3|} \rightarrow |2n+1|-|n+3| \geq 0 \Rightarrow |2n+1| \geq |n+3|$$

$$\frac{1}{2} \rightarrow 4n^2+4n+1 \geq n^2+4n+9 \Rightarrow 3n^2-2n-8 \geq 0$$

$$3n^2-2n-8 \geq 0 \Rightarrow n^2-2n-2\frac{8}{3} \geq 0 \Rightarrow (n-4)(n+\frac{4}{3}) \geq 0 \rightarrow n \geq \frac{4}{3} \text{ or } n \leq -\frac{4}{3}$$

$$D_f z (-\infty, -\frac{4}{3}] \cup [\frac{4}{3}, +\infty)$$

$$y z \log_p (1-\log_p^n) \rightarrow 1-\log_p^n > 0 \Rightarrow \log_p^n < 1 \Rightarrow \frac{n}{n} < p$$

$$\Rightarrow n < p$$

$$D_f z (-\infty, p)$$

$$y z \log_p (1-\log_p^{\frac{n}{p}}) \rightarrow 1-\log_p^{\frac{n}{p}} > 0 \Rightarrow \log_p^{\frac{n}{p}} < 1 \Rightarrow \frac{n}{p} < p$$

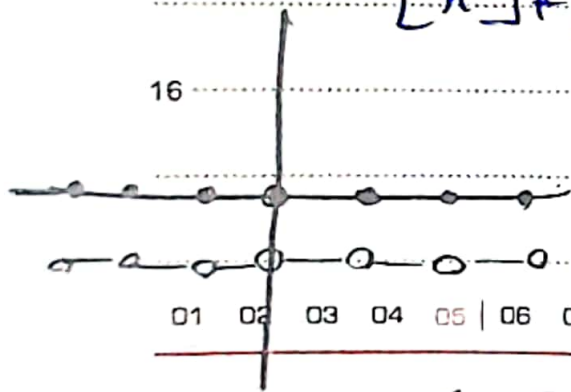
$$\Rightarrow n < p^2$$

$$D_f z (\frac{1}{p}, +\infty)$$

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$$f(n) = \frac{\sqrt{\epsilon - n^2}}{[n] + [-n] + 1} \quad \epsilon - n^2 \geq 0 \Rightarrow n^2 \leq \epsilon \Rightarrow -2 \leq n \leq 2 \quad (1)$$

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$$\Rightarrow n \in \mathbb{Z} \quad (2)$$

$$(1) \cap (2) \Rightarrow D_f = \{-2, -1, 0, 1, 2\}$$

$$f(n)$$

$$y[n] = [n] + [-n]$$

تشکیل جهاد دانشگاهی (۱۳۵۹-ه ش)

مرداد ۱۴۰۳

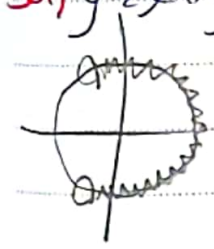
$$f(n) = \log \log \frac{(r_n-1)}{\omega} \rightarrow \log \frac{(r_n-1)}{\frac{1}{r}} \geq 0 \Rightarrow$$

$$\log \frac{r_n-1}{\omega} \leq 1 \Rightarrow r_n-1 \leq \omega \Rightarrow r_n \leq \omega+1 \Rightarrow n \leq r \quad (1) \quad \log \frac{(r_n-1)}{\omega} > 0 \Rightarrow r_n-1 > 1 \Rightarrow r_n > 2 \Rightarrow n > 1 \quad (2)$$

$$r_n-1 > 0 \Rightarrow r_n > 1 \Rightarrow n > \frac{1}{r} \quad (3) \quad \text{منطقه } D_f = (1, r]$$

$$y = \log(r \cos n + 1) \rightarrow r \cos n + 1 > 0 \Rightarrow r \cos n > -1 \Rightarrow \cos n > -\frac{1}{r} \quad \checkmark$$

$$D_f = (rk\pi - \frac{r\pi}{r}, rk\pi + \frac{r\pi}{r})$$



$$y = \sqrt{\log \frac{n-1}{n+1}} \rightarrow \log \left(\frac{n-1}{n+1} \right) \geq 0 \Rightarrow \frac{n-1}{n+1} \geq 1 \Rightarrow \frac{n-1}{n+1} - 1 \geq 0$$

$$\Rightarrow \frac{n-1-n-1}{n+1} \geq 0 \Rightarrow \frac{-2}{n+1} \geq 0 \Rightarrow \frac{-1}{\frac{1}{n+1}} \geq 0 \Rightarrow (-\infty, -1) \quad (4)$$

$$\frac{n-1}{n+1} > 0 \Rightarrow \frac{-1}{\frac{1}{n+1}} > 0 \Rightarrow (-\infty, -1) \cup (1, +\infty) \quad (5) \quad \text{منطقه } D_f = (-\infty, -1)$$

$$f(n) = \sqrt{(a+r)n^r + an + b} \quad D_f = (-\infty, r] \Rightarrow a = -r$$

با توجه به اینکه $a = -r$ و $r > 0$ داریم $a < 0$ و $b \geq 0$

$$\hookrightarrow an + b \geq 0 \Rightarrow -rn + b \geq 0 \Rightarrow b - r \geq 0 \Rightarrow b = r$$

$$f(n) = \sqrt{n^r + rn + r - m^r} \rightarrow n^r + rn + r - m^r \geq 0$$

از آنجایی که $D_f \subset \mathbb{R}$ یعنی مقادیر n و m در $\mathbb{R}$ قرار می‌گیرند و $a > 0$ و $r > 0$ داریم $n^r + rn + r - m^r \geq 0 \Rightarrow m^r \leq n^r + rn + r \Rightarrow m \leq n + 1$

اختلاف بین n و m حداکثر ۱ است $(1) = m$