

$$y = \frac{x^3 + 3x^2 - 1x + 3}{x^2 + 2x - 3} \quad (1)$$

$$\frac{(x+3) \rightarrow (-3)}{(x-1)(x-3)} = Df) \cdot R - \left\{ \frac{1}{x-1} + \frac{1}{x-3} \right\}$$

$$\rightarrow \frac{x^3 + 3x^2 - 1x + 3}{x^2 + 2x - 3} \rightarrow x + 1 + \frac{(x+1)}{(x-1)(x-3)} \rightarrow \frac{(x+3)^{-1}}{(x-1)(x-3)(x+1)}$$

$$Df) \cdot R - \left\{ -\frac{1}{x-1} + \frac{1}{x-3} \right\} \rightarrow -1$$

$$y = \frac{x^3 + 3x^2 - 1x + 3}{x^2 + 2x - 3} \rightarrow \frac{x^3 - 2x^2 + 3x + 1}{x^2 + 2x - 3} \rightarrow \frac{(x+3)}{(x-1)(x-3)}$$

$$Df) \cdot R - \{1\}$$

$$y = \sqrt{\frac{(x+3) \rightarrow (-3)}{x^2 + 2x - 3}} \rightarrow \frac{x^3 - 2x^2 + 3x + 1}{x^2 + 2x - 3} \rightarrow \frac{(x+3)}{(x-1)(x-3)}$$

$$Df) \cdot R - \left\{ \frac{1}{x-1} + \frac{1}{x-3} \right\}$$

$$y = \frac{x}{x^2 + 2x - 3} \rightarrow \frac{x}{(x-1)(x-3)}$$

$$\textcircled{1} x > 1 \rightarrow x^2 - 2x + 3 - 2x + 3 \rightarrow x^2 - 4x + 6$$

$$\textcircled{2} x < 1 \rightarrow x^2 + 2x - 3 - 2x + 3 \rightarrow x^2 - 4x + 6$$

$$\textcircled{3} x < 1 \rightarrow x^2 + 2x - 3 - 2x + 3 \rightarrow x^2 - 4x + 6$$

$$\textcircled{1} \cup \textcircled{2} \cup \textcircled{3} \Rightarrow Df) \cdot R - \left\{ -\frac{1}{x-1} + \frac{1}{x-3} \right\}$$

$$y = \frac{x+3}{|x+1| - |x+3|}$$

$$|x+1| - |x+3| \neq 0 \rightarrow |x+1| = |x+3|$$

$$(x+1)^2 = (x+3)^2 \rightarrow x^2 + 2x + 1 = x^2 + 6x + 9 \rightarrow 2x + 1 = 6x + 9 \rightarrow -4x = 8 \rightarrow x = -2$$

$$Df) \cdot R - \left\{ \frac{1}{x-1} + \frac{1}{x-3} \right\}$$

$$\Rightarrow y = \sqrt{|x+1| - |x-1|} \rightarrow |x+1| - |x-1| \geq 0 \rightarrow |x+1| \geq |x-1|$$

$$(x+1)^2 \geq (x-1)^2 \rightarrow x^2 + 2x + 1 \geq x^2 - 2x + 1$$

$$4x \geq 0 \rightarrow x \geq 0$$

$$\frac{x}{x+1} \geq \frac{x}{x-1}$$

$$\left[-\frac{1}{2} - \frac{1}{2} \right] \cup [1, +\infty)$$

$$\Leftrightarrow (x-1)(x+1) \geq 0$$

$$y = \log(1 - \log_x^a)$$

$$\textcircled{1} \wedge \textcircled{2} \Rightarrow (0, 5]$$

(a)

$$\textcircled{1} x > 0 \quad \textcircled{2} 1 - \log_x^a > 0 \rightarrow 1 > \log_x^a \quad x > x$$

$$\Rightarrow y = \log_x(1 - \log_x^a) \rightarrow \textcircled{1} x > 0$$

$$\textcircled{2} 1 - \log_x^a > 0 \rightarrow 1 > \log_x^a$$

$$D_f = \textcircled{1} \wedge \textcircled{2} \Rightarrow \left(\frac{1}{e}, +\infty \right)$$

$$\frac{1}{e} < x$$

$$f(x) = \sqrt{\log_{x+1}^{\log_{x-1}^a}} \Rightarrow \textcircled{1} x+1 > 0 \rightarrow x > -1$$

(b)

$$\textcircled{2} \log_{x+1}^{\log_{x-1}^a} > 0 \rightarrow x+1 > 1 \rightarrow x > 0$$

$$\textcircled{3} \log_{x+1}^{\log_{x-1}^a} \geq 0 \rightarrow \log_{x+1}^{\log_{x-1}^a} \leq 1 \rightarrow x+1 \leq a$$

$$\Leftrightarrow x \leq a-1$$

$$x \leq 3$$

$$D_f = \textcircled{1} \wedge \textcircled{2} \wedge \textcircled{3} \Rightarrow (1, 3]$$

$$\Rightarrow y = \log(x \cos x + 1) \quad x \cos x + 1 > 0 \rightarrow x \cos x > -1$$

(c)

$$D_f = \left(x \cos x - \frac{1}{x} > 0, x \cos x + \frac{1}{x} > 0 \right)$$

$$\Rightarrow y = \sqrt{\log \frac{x-1}{x+1}}$$

$$\textcircled{1} \frac{x-1}{x+1} > 0 \rightarrow \frac{-1}{x+1} > 0$$

$$\textcircled{2} \log \frac{x-1}{x+1} \geq 0 \rightarrow \frac{x-1}{x+1} \geq 1 \rightarrow \frac{x-1}{x+1} > 1$$

$$\frac{-x}{x+1} > 0 \rightarrow \frac{-1}{x+1} > 0$$

$$D_f = \textcircled{1} \wedge \textcircled{2} \Rightarrow (-\infty, -1)$$

$$f(n) = \sqrt{(a+r)n^2 + an + b} \rightarrow (-\infty, \infty] \rightarrow \text{فريد} \quad b = ?$$

$$\hookrightarrow a = -r \rightarrow \sqrt{-rn + b} \rightarrow -rn + b \geq 0 \rightarrow -ra \geq b$$

$$\frac{b}{r} \geq r \rightarrow \boxed{b \geq r^2} \quad n \leq \frac{b}{r} \rightarrow \boxed{D = (-\infty, r]}$$

$$f(n) = \sqrt{n^2 + rn + r - n^2}$$

$$\delta \leq 0 \rightarrow b^2 - 4ac \leq 0$$

$$\hookrightarrow f(r - m^2) \leq 0$$

$$-1 \leq m \leq 1$$

$$\text{فريد} \rightarrow 1 - (-1) \geq r$$

$$\leftarrow f - \lambda a \leq m^2 \rightarrow f m^2 \leq f$$

$$f(n) = \frac{\sqrt{f - n^2}}{[n] + [-n] + 1}$$

$$(i) f - n^2 \geq 0 \rightarrow f \geq n^2 \rightarrow -r \leq n \leq r$$

$$(ii) [n] + [-n] + 1 \neq 0 \rightarrow [n] + [-n] \neq -1$$

$$D_f \cap D_{f'} \cap D_{f''} \rightarrow \{-2, -1, 0, 1, 2\}$$

$$\begin{cases} [n] + [-n] < 0 \rightarrow n \in \mathbb{Z}^+ \\ [n] + [-n] = -1 \rightarrow n \notin \mathbb{Z}^+ \end{cases}$$

$$\text{مجموعه } D_f(D)$$