① دامنه توابع زیر را بدست آورید

الف) $y = \frac{x+3}{2x^3+3x^2-8x+3}$

مخرج ضرایب صفر $\neq 0$

$2x^3+3x^2-8x+3 \mid x-1$

$2x^3+5x^2-3$

$-3x^3+2x^2$

$5x^2$

$-5x^2+5x$

$3x+3$

$-3x+3$

0

$2x^3+5x^2-3 = (x-1)(2x^2+5x-3)$

$\Delta = 25+24 = 49$

$x = \frac{-5 \pm 7}{4}$

$x = \frac{1}{2}$, $x = -3$

$D_f = \mathbb{R} - \{1, \frac{1}{2}, -3\}$

ب) $\frac{x+3}{2x^3+7x^2+10x+3} = \frac{x+3}{(x+1)(2x^2+5x+3)} \neq 0$

مخرج ضرایب مجموع صفر

$2x^3+7x^2+10x+3 \mid x+1$

$2x^3+5x^2+3$

$2x^2-2x^2$

$7x^2+10x$

$7x^2+7x$

$3x+3$

$-3x-3$

0

$(x+1)(2x^2+5x+3) = 0$

$\Delta = 25-24 = 1$

$x = \frac{-5 \pm 1}{4}$

$x = -1$, $x = -\frac{3}{2}$

$D_f = \mathbb{R} - \{-1, -\frac{3}{2}\}$

② الف) $y = \frac{x+3}{x^3-2x^2+2x-1}$

مخرج ضرایب صفر $\neq 0$

$x^3-2x^2+2x-1 \mid x-1$

x^3-x^2+1

$-x^2+2x-1$

$-x^2+x$

$x-1$

$-x+1$

0

$(x-1)(x^2-x+1) = 0$

$\Delta = 1-4 = -3 < 0$

ریشه ندارد

$D_f = \mathbb{R} - \{1\}$

ب) $\sqrt{\frac{x+3}{x^3-2x^2+1}}$

ریشه ما قدمت الف ۱ است

	-3		1	
	$+$	0	$-$	$+$

$D_f = (-\infty, -3] \cup (1, +\infty)$

$$y=\frac{2}{x^2-5|x-1|-2x+5}\neq 0 \longrightarrow x^2-5x+5-2x+5\neq 0$$

$$x^2+5x-5-2x+5\neq 0$$

$$\longrightarrow x^2-7x+10\neq 0 \longrightarrow (x-2)(x-5) \longrightarrow x_1=2,\ x_2=5$$

$$x^2+3x\neq 0 \longrightarrow x(x+3)\neq 0 \longrightarrow x_3=0,\ x_4=-3$$

$$\longrightarrow D_f=\mathbb{R}-\{-3,0,2,5\}$$

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④ الف)

$$y=\frac{x+3}{|2x+1|-|x+3|}\neq 0 \longrightarrow |2x+1|=|x+3| \xrightarrow{\text{توان ۲}} 4x^2+4x+1 \neq x^2+6x+9$$

$$3x^2-2x-8\neq 0 \longrightarrow \frac{2\pm 10}{6} \begin{cases} 2 \\ -\frac{4}{3}\end{cases} \longrightarrow D_f=\mathbb{R}-\left\{-\frac{4}{3},2\right\}$$

$$\Delta=4-(4\times 3\times -8)=100$$

ب)

$$y=\sqrt{|2x+1|-|x+3|}\geq 0 \longrightarrow |2x+1|\geq |x+3| \xrightarrow{\text{توان ۲}} 4x^2+4x+1\geq x^2+6x+9$$

$$\longrightarrow 3x^2-2x-8\geq 0 \longrightarrow \Delta=100 \longrightarrow \frac{2\pm 10}{6} \begin{cases} 2 \\ -\frac{4}{3}\end{cases}$$

	$-\frac{4}{3}$		2	
$+$	0	$-$	0	$+$

$$\longrightarrow D_f=\left(-\infty,-\frac{4}{3}\right]\cup[2,+\infty)$$

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⑤

الف) $y=\log_2(1-\log_3 x) \longrightarrow 1-\log_3 x>0 \longrightarrow \begin{cases} x>0 \\ \log_3 x<1 \longrightarrow x<3\end{cases}$

$$D_f=(0,3)$$

ب) $\log_2(1-\log_{\frac{1}{2}} x) \longrightarrow 1-\log_{\frac{1}{2}} x>0 \quad \begin{cases} x>0 \\ \log_{\frac{1}{2}} x<1 \longrightarrow x>\frac{1}{2}\end{cases}$

$$D_f=\left(\frac{1}{2},+\infty\right)$$

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⑥

$$f(x)=\sqrt{\log_{0/5}\log_5(2x-1)} \longrightarrow \log_{0/5}\log_5(2x-1)\geq 0 \longrightarrow \log_5(2x-1)>0 \begin{cases} 2x-1>0 \ ① \\ \longrightarrow x>\frac{1}{2} \\ 2x-1>1 \longrightarrow x>1 \ ②\end{cases}$$

$$\log_5(2x-1)\leq 1 \longrightarrow 2x-1\leq 5 \longrightarrow 2x\leq 6 \longrightarrow x\leq 3$$

$$\xrightarrow{③,②,①} D_f=(1,3]$$

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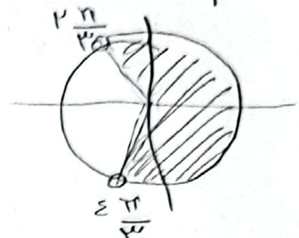
الف) $y = \log(2\cos x + 1) \to 2\cos x + 1 > 0 \to 2\cos x > -1$

$\cos x > -\frac{1}{2}$

$D_f = (2k\pi - \frac{2\pi}{3}, 2k\pi + \frac{2\pi}{3})$



ب) $\sqrt{\log \frac{x-1}{x+1}} \geq 0 \to \frac{x-1}{x+1} \geq 1 \to \frac{x-1}{x+1} - 1 \geq 0 \to \frac{x-1-x-1}{x+1} \geq 0$

$\to \frac{-2}{x+1} \geq 0$ جواب $(-\infty, -1)$ ①

$D_f = (-\infty, -1)$

$\frac{x-1}{x+1} > 0 \to (-\infty, -1) \cup (1, +\infty)$ ②

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$f(x) = \sqrt{(a+2)x^2 + ax + b}$ → دامنه تعریف $(-\infty, 3]$ → عبارت درجه اول و ضریب x^2 صفر

$\to a+2=0 \to a=-2$

$? = b$

$-2x + b \geq 0 \xrightarrow[y=0]{x=3} b - 6 = 0 \to b = 6$

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$f(x) = \sqrt{x^2 + 2x + 2 - m^2} \to D_f = \mathbb{R}$

$m_{max} - m_{min} = ?$

$x^2 + 2x + 2 - m^2 \geq 0$

$\to D_f = \mathbb{R} \to \Delta = 0,\ a > 0 \to 4 - 8 + 4m^2 = 0 \to 4m^2 = 4 \to m^2 = 1 \to m = \pm 1$

$m_{max} - m_{min} = 1 - (-1) = 2$

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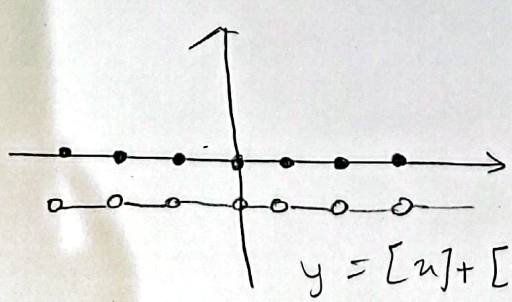
$f(x) = \frac{\sqrt{4 - x^2}}{[x] + [-x] + 1} \to 4 - x^2 \geq 0 \to x^2 \leq 4 \to -2 \leq x \leq 2$ ①

$[x] + [-x] + 1 \neq 0 \to [x] + [-x] \neq -1$

$\to x \in \mathbb{Z}$ ②

①,② $\to D_f = \{-2, -1, 0, 1, 2\}$



$y = [x] + [-x]$

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