تکلیف شماره 28 ۲۰ هستی باقری یازدهم

①

$$\frac{1}{\sqrt{\cos^2\alpha}}-\frac{1}{\cot\alpha}=\frac{1-\sin\alpha}{|\cos\alpha|}\Rightarrow\frac{1}{|\cos\alpha|}-\tan\alpha=\frac{1-\sin\alpha}{|\cos\alpha|}$$

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$$\frac{1-1+\sin\alpha}{|\cos\alpha|}=\frac{\sin\alpha}{\cos\alpha}\Rightarrow|\cos\alpha|=\cos\alpha$$

$\cos$ و $\sin\alpha$ مثبت هستند → ناحیه اول

$$\cot\alpha=\frac{\cos\alpha}{\sqrt{\underbrace{1-\cos^2\alpha}_{\sin^2\alpha}}}\Rightarrow\frac{\cos\alpha}{\sin\alpha}=\frac{\cos\alpha}{|\sin\alpha|}\Rightarrow|\sin\alpha|=\sin\alpha$$

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②

$$\frac{-\pi}{12}<x<\frac{5\pi}{12}\to\frac{-\pi}{6}<2x<\frac{5\pi}{6}\to\frac{-1}{2}<\sin 2x\leq 1$$

$$\frac{-1}{2}<\frac{m-1}{4}\leq 1\to -2<m-1\leq 4\to -1<m\leq 5$$

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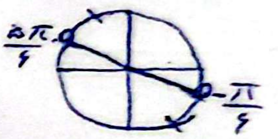

مجموعه مقادیر m $\Rightarrow(-1,5]$

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③

$$\tan x+\cot x=-3\to\frac{\overbrace{\sin^2x+\cos^2x}^{1}}{\sin x\cos x}=-3\to\sin x\cos x=\frac{-1}{3}$$

$$\sin^3x+\cos^3x=(\sin x+\cos x)(\underbrace{\sin^2x+\cos^2x}_{1}-\underbrace{\sin x\cos x}_{\frac{-1}{3}})=\frac{-1}{\sqrt{3}}\times\frac{4}{3}=\frac{-4}{3\sqrt{3}}$$

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قبول

$$(\sin x+\cos x)^2=\sin^2x+\cos^2x+2\sin x\cos x=1-\frac{2}{3}=\frac{1}{3}\to\sin x+\cos x=\pm\frac{1}{\sqrt{3}}$$

$$3\pi<4x<4\pi\to\frac{3\pi}{4}<x<\pi\to-1<\cos x<\frac{-\sqrt{2}}{2},\ 0<\sin x<\frac{\sqrt{2}}{2}$$

$$\to\sin x+\cos x<0\qquad\frac{1}{\sin^3x+\cos^3x}=\frac{1}{\frac{-4}{3\sqrt{3}}}=\frac{-3\sqrt{3}}{4}$$

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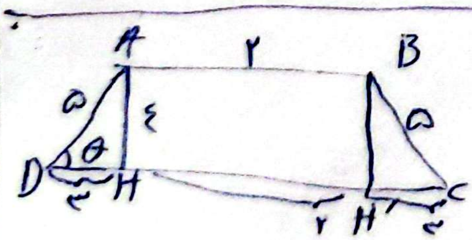
④



$$\cos\theta=\frac{DH}{AD}=\frac{DH}{5}=\frac{6}{10}\Rightarrow DH=3$$

$$\to CH'=3\qquad AB=HH'=2$$

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$$\to CD=3+3+2=8$$

$$AD^2=DH^2+AH^2$$
$$5^2=3^2+AH^2$$
$$AH=4$$

$$S_{\text{ذوزنقه}}=\frac{(AB+CD)\times AH}{2}=\frac{(2+8)\times 4}{2}=20$$

(5)

$$\tan(285^\circ)\tan(-165) - \sin(1095^\circ)\cos(255^\circ)$$

$$= \tan\left(\frac{3\pi}{2}+15\right) \times \tan(-\pi+15) - \sin(6\pi+15) \times \cos\left(\frac{3\pi}{2}-15\right)$$

$$= -\cot 15 \times \tan 15 - \sin 15 \times -\sin 15 = -1 + \sin^2 15$$

$$= -1 + (1-\cos^2 15) = -\cos^2 15 = k\cos^2 15 \Rightarrow \boxed{k=-1}$$

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(6)

$$A = \sqrt{3}\cos(210) \times \sin(243) - \sqrt{2}\sin(135)\cos(153)$$

$$= \sqrt{3} \times \frac{-\sqrt{3}}{2} \times \sin\left(\frac{3\pi}{2}-27\right) - \sqrt{2} \times \frac{\sqrt{2}}{2} \times \cos(\pi-27)$$

$$= \frac{-3}{2} \times -\cos 27 - 1 \times -\cos 27 = \frac{3}{2}\cos 27 + \cos 27 = \frac{5}{2}\cos 27$$

$$\frac{A}{\cos 27^\circ} = \frac{\frac{5}{2}\cos 27^\circ}{\cos 27^\circ} = \frac{5}{2}$$

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(7)

$$f\left(\frac{\pi}{24}\right) = 16\cos^2\left(\frac{\pi}{12}\right) \times \underbrace{\cos^2\left(\frac{\pi}{6}\right)}_{\frac{1}{4}} \times \underbrace{\cos^2\left(\frac{\pi}{3}\right)}_{\frac{3}{4}} \times \underbrace{\cos^2\left(\frac{2\pi}{3}\right)}_{\frac{1}{4}}$$

$$= \cancel{16} \times \frac{2+\sqrt{3}}{4} \times \frac{1}{\cancel{4}} \times \frac{3}{4} \times \frac{1}{\cancel{4}} = \frac{6+3\sqrt{3}}{16}$$

$$\cos^2\left(\frac{\pi}{12}\right) = \frac{1+\cos\frac{\pi}{6}}{2} = \frac{1+\frac{\sqrt{3}}{2}}{2} = \frac{\frac{2+\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}}{4}$$

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(8)

$$\frac{1-\sin x}{1+\sin x} = 4 \rightarrow 1-\sin x = 4+4\sin x \rightarrow 5\sin x = -3$$

$$\sin x = \frac{-3}{5}$$

$$\sin^2 x + \cos^2 x = 1 \rightarrow \frac{9}{25} + \cos^2 x = 1 \rightarrow \cos x = \frac{-4}{5} \checkmark$$

$$\cos x = \frac{4}{5}$$ غ ق ق

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{\frac{-3}{5}}{1+\frac{-4}{5}} = \frac{\frac{-3}{5}}{\frac{1}{5}} = -3$$

Scanned with CamScanner

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$$\frac{\sin\theta}{۱-\cos\theta}+\frac{۱+\cos\theta}{\sin\theta}\xrightarrow{\text{با توجه به اینکه}}\frac{\sin\theta}{۱+\cos\theta}=\frac{۱-\cos\theta}{\sin\theta}=\tan\frac{\theta}{۲}$$

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$$=\cot\frac{\theta}{۲}+\cot\frac{\theta}{۲}=۲\cot\frac{\theta}{۲}=k\cot\frac{\theta}{۲}\Longrightarrow \boxed{k=۲}$$

۱۰

$$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right)=\underbrace{\cos\frac{۱۱\pi}{۴}}_{\frac{-\sqrt{۲}}{۲}}\cos\alpha-\underbrace{\sin\frac{۱۱\pi}{۴}}_{\frac{\sqrt{۲}}{۲}}\sin\alpha$$

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$$=\frac{-\sqrt{۲}}{۲}\times\frac{-۷\sqrt{۲}}{۱۰}-\frac{\sqrt{۲}}{۲}\times\frac{\sqrt{۲}}{۱۰}=\frac{۷}{۱۰}-\frac{۱}{۱۰}=\frac{۶}{۱۰}=\frac{۳}{۵}$$

$$\sin^۲\alpha+\cos^۲\alpha=۱\longrightarrow\frac{۲}{۱۰۰}+\cos^۲\alpha=۱$$

$$\cos\alpha=+\frac{۷\sqrt{۲}}{۱۰}\quad \text{غ ق ق}$$

$$\cos\alpha=\frac{-۷\sqrt{۲}}{۱۰}\quad ✓$$