باران کاظمی یازدهم C - تکلیف شماره ی ۲۸

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$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\frac{-1-\sin\alpha}{\cos\alpha} = \left\langle \begin{array}{l} \cancel{\frac{1-\sin\alpha}{\cos\alpha}} \\ \frac{-1+\sin\alpha}{\cos\alpha} \end{array}\right.$$

$$\rightarrow \frac{1-\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \rightarrow \boxed{\cos\alpha > 0} \quad ①$$

$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \rightarrow \boxed{\sin\alpha > 0} \quad ②$$

$\xrightarrow{②,①}$ ناحیه اول

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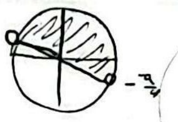
$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6}$$

$$-\frac{1}{2} < \sin 2x \le 1 \rightarrow -\frac{1}{2} < \frac{m-1}{4} \le 1$$

$$-2 < m-1 \le 4 \rightarrow \boxed{-1 < m \le 5}$$



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$$\frac{1}{\sin^3x+\cos^3x} = \frac{1}{(\underbrace{\sin x+\cos x}_{\sqrt{\frac13}})(\underbrace{\sin^2x+\cos^2x-\sin x\cos x}_{1+\frac13=\frac43})} = \frac{1}{\sqrt{\frac13}\left(\frac43\right)} = \frac{1}{\frac{\sqrt3}{3}\times\frac43} = \frac{-9}{4\sqrt3} = \frac{9\sqrt3}{3\times4} = \boxed{\frac{3\sqrt3}{4}}$$

$$\tan x+\cot x=-3 \rightarrow \frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=-3 \rightarrow \frac{\cancel{\sin^2x+\cos^2x}}{\sin x\cos x}=-3 \rightarrow \sin x\cos x=-\frac13$$

$$(\sin x+\cos x)^2 = \underbrace{\cancel{\sin^2x+\cos^2x}}_{1} + \underbrace{2\sin x\cos x}_{-\frac23} = \frac13 \rightarrow \sin x+\cos x=\sqrt{\frac13}$$

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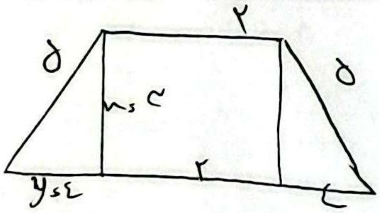

$$\frac{h}{5}=0{,}6 \rightarrow h=3$$

$$\sin\theta = 1-0{,}36 = 0{,}64 \rightarrow \sin\theta = 0{,}8$$

$$\frac{y}{5}=0{,}8 \rightarrow y=4$$

$$S=\frac{3\times12}{2}=\boxed{18}$$

$$\cos\theta=\frac{h}{5}=\frac{6}{10} \quad h=3$$

$$\sin\theta=\frac{h}{5}=\frac{8}{10} \quad h=4$$

$$S=\frac{(2+8)}{2}\times4=20$$

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(۵

$$\tan(285^\circ)\tan(-195)-\sin(1095)\cos(255)$$

$$-\tan(285)\tan(195)-\sin 1095\cos(255)$$

$$-\tan\left(\frac{3\pi}{2}+15\right)\tan(\pi-15)-\sin(6\pi+15)\cos\left(\frac{3\pi}{2}-15\right)$$

$$-\cancel{\cot 15}\,\cancel{\tan 15}+\sin 15\sin 15 = \sin^2 15-1 = -\cos^2 15 = k\cos^2 15 \rightarrow \boxed{k=-1}$$

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$$\sqrt3\cos(210)\sin(243)-\sqrt2\sin(135)\cos(153)$$

$$=\sqrt3\cos(210)\sin\left(\frac{3\pi}{2}-27\right)-\sqrt2\,(\sin(135))\,(\cos(\pi-27))$$

$$=\sqrt3\left(-\frac{\sqrt3}{2}\right)(-\cos27)-(\sqrt2)\left(\frac{\sqrt2}{2}\right)(-\cos27)$$

$$=\frac32\cos27+\cos27=\frac52\cos27 \rightarrow \frac{\frac52\cos27}{\cos27}=\boxed{\frac52}$$

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$f(x) = 16\left[\frac{\sin^2(2x)}{\sin^2(2x)}\cos^2(2x)\cos^2(4x)\cos^2(12x)\cos^2(24x)\right]$ (۷

$4\sin^2(4x)$, $4\sin^2(12x)$, $4\sin^2(24x)$, $4\sin^2(48x)$

$$\frac{\cancel{16}\,\sin^2\left(\frac{48\pi}{24}\right)}{\cancel{4}\times4\times4\times4\,\sin^2\left(\frac{2\pi}{24}\right)} = \frac{\frac{3}{4}}{4\times4\times\frac{2-\sqrt3}{4}} = \frac{3}{16(2-\sqrt3)}\times\frac{(2+\sqrt3)}{(2+\sqrt3)} = \boxed{\frac{6+3\sqrt3}{16}}$$ ۲

$$\sin^2\left(\frac{\pi}{12}\right) = \frac{1-\cos\left(\frac{\pi}{6}\right)}{2} = \frac{1-\frac{\sqrt3}{2}}{2} = \frac{2-\sqrt3}{4}$$

$$\frac{1-\sin x}{1+\sin x} = 4 \to 1-\sin x = 4+4\sin x \to 5\sin x = -3 \to \sin x = -\frac{3}{5}$$ (۸

$\cos x = -\frac{4}{5}$ ربع سوم $\qquad \tan x = \frac{-\frac35}{-\frac45} = \frac{3}{4}$ ۱,۷۵

$$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \to \frac{3}{4} = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \to 3-3\tan^2\frac{x}{2} = 8\tan\frac{x}{2}$$

$$3\tan^2\frac{x}{2} + 8\tan\frac{x}{2} - 3 = 0 \to \frac{-8\pm\sqrt{\overset{10}{64+36}}}{6}$$

-3 قبول ✓ $\to \boxed{\tan\frac{x}{2} = \frac{1}{3}}$

$\frac{1}{3}$ ✗

$$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} \to \frac{\sin^2\theta + 1-\cos^2\theta}{\sin\theta(1-\cos\theta)} = \frac{(\sin^2\theta+\sin^2\theta+\cancel{\cos^2\theta}-\cancel{\cos^2\theta})}{\sin\theta(1-\cos\theta)}$$ (۹

$$\Rightarrow \frac{2\sin^2\theta}{\cancel{\sin\theta}(1-\cos\theta)} \to \frac{2\sin\theta}{1-\cos\theta} = \frac{4\cancel{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}}{\sin^{\cancel{2}}\frac{\theta}{2}} = 4\cot\left(\frac{\theta}{2}\right)$$ ۱

$\boxed{k=4}$

$$\cos\left(\frac{11\pi}{4}+\alpha\right) = \cos\frac{11\pi}{4}\cos\alpha - \sin\frac{11\pi}{4}\sin\alpha$$ (۱۰

ربع

$$= \left(-\frac{\sqrt2}{2}\right)\left(-\frac{7\sqrt2}{10}\right) - \left(\frac{\sqrt2}{2}\right)\left(\frac{\sqrt2}{10}\right) = \frac{7}{10} - \frac{1}{10} = \frac{6}{10} = \boxed{\frac{3}{5}}$$ ۲

$$\cos\frac{11\pi}{4} = \underbrace{\cos\left(\pi + \frac{7\pi}{4}\right)}_{\cos\pi\cos\frac{7\pi}{4} - \underbrace{\sin\pi\sin\frac{7\pi}{4}}_{0}} = -\frac{\sqrt2}{2}$$

$-1\left(\frac{\sqrt2}{2}\right)$

$$\cos\alpha = -\frac{7\sqrt2}{10}$$

۹) $\frac{\sin^2\theta+(1-\cos^2\theta)}{(1-\cos\theta)\sin\theta} = \frac{2\sin^2\theta}{\sin\theta(1-\cos\theta)} = \frac{2\times2\times\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = 2\cot\frac{\theta}{2}$

$\to k=2$