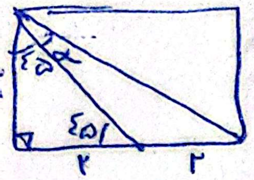
تکلیف شماره ۲۷ ‏ ۲۰

حسن بابایی - یازدهم ریاضی

(۱)

$$4/5 = \frac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha \Longrightarrow \sin\alpha = \frac{\sqrt{3}}{2} \left\langle \begin{array}{l} \alpha = 120 \\ \alpha = 60 \end{array} \right. \quad \frac{120}{60} = 2$$

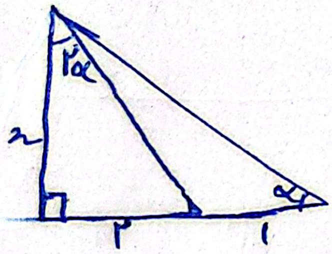
(۲)



$$\cot(\alpha + 45) = \frac{\cot\alpha \cot 45^\circ - 1}{\cot 45^\circ + \cot\alpha} = \frac{2}{4} = \frac{1}{2}$$

$$\Rightarrow \frac{\cot\alpha - 1}{1 + \cot\alpha} = \frac{1}{2} \Rightarrow 2\cot\alpha - 2 = 1 + \cot\alpha \Rightarrow \boxed{\cot\alpha = 3}$$

(۳)



$$\cot\alpha = \frac{3}{x} = ?$$

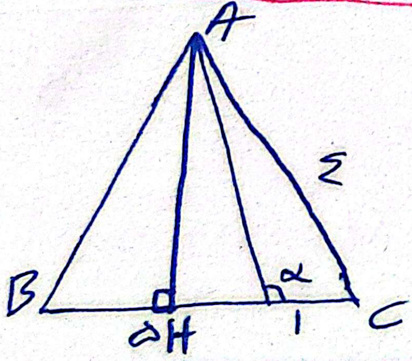
$$\tan\alpha = \frac{x}{3} \qquad \tan 2\alpha = \frac{2}{x}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha} = \frac{\frac{2x}{3}}{\frac{9 - x^2}{9}} = \frac{2}{x}$$

$$\Rightarrow \frac{6x}{9 - x^2} = \frac{2}{x} \Rightarrow 6x^2 = 18 - 2x^2 \Rightarrow 8x^2 = 18$$

$$\Longrightarrow x = \pm\frac{3}{2} \left\langle \begin{array}{l} x = -\frac{3}{2} \text{ غ ق ق} \\ x = \frac{3}{2} \checkmark \end{array} \right. \quad \tan\alpha = \frac{\frac{3}{2}}{3} = \frac{1}{2} \Rightarrow \boxed{\cot\alpha = 2}$$

(۴)



$$\overset{\triangle}{AHC} \Rightarrow AH^2 + 9 = 16 \rightarrow AH = \sqrt{7}$$

$$\overset{\triangle}{AEH} \Rightarrow \tan(180 - \alpha) = \frac{\sqrt{7}}{2}$$

$$\tan(180 - \alpha) = -\tan\alpha \Longrightarrow \tan\alpha = -\frac{\sqrt{7}}{2}$$

(۵)

$$2\sin^2 x + \cos^2 x = \frac{4}{3} \Rightarrow \sin^2 x + 1 = \frac{4}{3} \Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \sin x = \pm\frac{1}{\sqrt{3}}$$

$$\cos\alpha = \pm\frac{\sqrt{2}}{\sqrt{3}} \qquad \boxed{\tan^2 x = \frac{1}{2}}$$

(۶)

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1 + \cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1 + \sin^2\alpha} = \frac{\sin^4\alpha + 4(1 - \sin^2\alpha)}{1 + (1 - \sin^2\alpha)} - \frac{\cos^4\alpha + 4(1 - \cos^2\alpha)}{1 + (1 - \cos^2\alpha)}$$

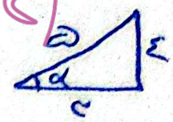
$$= \frac{\sin^4\alpha - 4\sin^2\alpha + 4}{2 - \sin^2\alpha} - \frac{\cos^4\alpha - 4\cos^2\alpha + 4}{2 - \cos^2\alpha} = \frac{(2 - \sin^2\alpha)^2}{2 - \sin^2\alpha} - \frac{(2 - \cos^2\alpha)^2}{2 - \cos^2\alpha}$$

$$= 2 - \sin^2\alpha - 2 + \cos^2\alpha = \cos^2\alpha - \sin^2\alpha = \boxed{\cos 2\alpha}$$

(۷)

$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)-\tan\left(\alpha-\frac{3\pi}{2}\right) = \cos\alpha \times -\sin\alpha + \cot\alpha$

$\Rightarrow \left(\frac{-3}{5}\times\frac{4}{5}\right)+\frac{3}{4} = \frac{-48+75}{100} = \frac{27}{100}$



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(۸)

$3\cos 4x+\sqrt{2}\sin x-\sqrt{2}\cos x = 3\cos\left(\frac{\pi}{3}\right)+\sqrt{2}(\sin x-\cos x)$

$\sin x-\cos x=\sqrt{2}\sin\left(x-\frac{\pi}{4}\right) \Rightarrow \frac{3}{2}+\sqrt{2}\times\sqrt{2}\times\sin\left(\frac{\pi}{12}-\frac{\pi}{4}\right)$

$= \frac{3}{2}+2\sin\left(\frac{-\pi}{6}\right) = \frac{3}{2}-1 = \frac{1}{2}$

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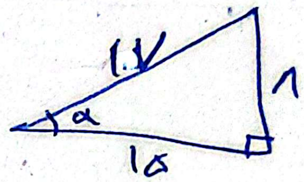
(۹)

$\tan\alpha = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)} = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$



$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha} = \frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}} = \frac{-16}{105}$

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(۱۰)

$\frac{\cot\alpha}{\sin\alpha}>0 \longrightarrow \frac{\cos\alpha}{\sin^2\alpha}>0 \longrightarrow \cos\alpha>0$ ①

$\sin 2\alpha = 2\sin\alpha\cos\alpha \longrightarrow 2\sin\alpha<\sin 2\alpha \longrightarrow 2\sin\alpha<2\sin\alpha\cos\alpha$

$\xrightarrow{\sin\alpha>0} \cos\alpha>1 \Rightarrow$ غ ق ق

$\xrightarrow{\sin\alpha<0} \cos\alpha<1$ ق ق ②

①, ② $\Rightarrow$ ربع چهارم
