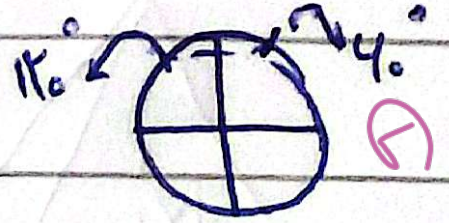
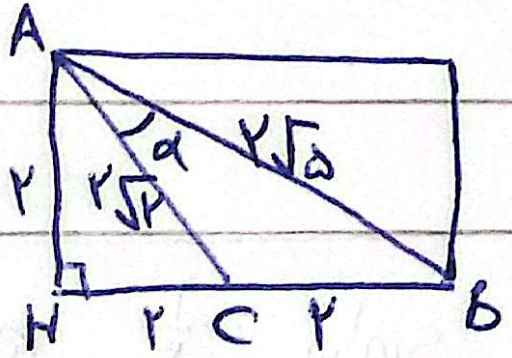
ریاضی سال نظری — تکلیف کد ۲۷ — یازدهم زمستان

$$S=\frac{1}{2}\times\sqrt{3}\times 4\times\sin\alpha=4/\omega \rightarrow \sin\alpha=\frac{\sqrt{3}}{2}$$



① ۲

$$\frac{120}{60}=2 \quad \text{میدانیم}$$



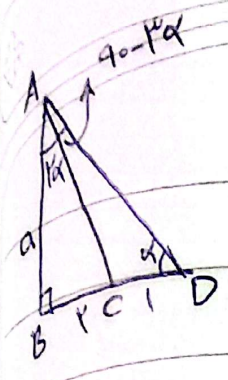
$$S_{ABC}=\frac{BC\times AH}{2}=\frac{1}{2}\times AC\times AB\times\sin\alpha \quad ②$$

$$\frac{1}{2}\times 2\sqrt{2}\times 2\sqrt{5}\times\sin\alpha=4 \rightarrow 2\sqrt{10}\sin\alpha=4 \quad ③$$

$$\sin\alpha=\frac{\sqrt{10}}{10}$$

$$\sin^2\alpha+\cos^2\alpha=1 \rightarrow \cos^2\alpha=\frac{9}{10} \rightarrow \cos\alpha=\frac{3\sqrt{10}}{10}$$

$$\cot\alpha=\frac{\cos\alpha}{\sin\alpha}=\frac{\frac{3\sqrt{10}}{10}}{\frac{\sqrt{10}}{10}}=3$$



$AC=\sqrt{a^2+4}$

$AD=\sqrt{a^2+9}$

$\frac{S_{ABC}}{S_{ACD}}=\frac{BC}{CD}=\frac{2}{1}=\frac{a\sqrt{a^2+4}\,2\sin\alpha\cos\alpha}{\sqrt{a^2+9}\sin\alpha}$ (۳)

$\cos\alpha=\frac{\sqrt{a^2+9}}{a\sqrt{a^2+4}} \leftarrow a\sqrt{a^2+4}\cos\alpha=\sqrt{a^2+9}$

$\cos\alpha=\frac{3}{\sqrt{a^2+9}}=\frac{\sqrt{a^2+9}}{a\sqrt{a^2+4}} \rightarrow a^2+9=3a\sqrt{a^2+4}$

$\sqrt{a^2+4}=\frac{a}{3}+\frac{3}{a} \rightarrow a^2+4=\frac{a^2}{9}+\frac{9}{a^2}+6$

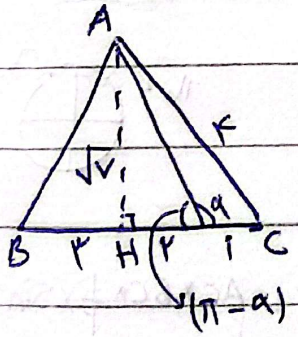
$\frac{8a^2}{9}+2-\frac{9}{a^2}=0$

$8a^4+18a^2-81=0 \leftarrow \frac{8a^4}{9}+2a^2-9=0$

$x=\frac{-18+36}{16}=\frac{9}{4} \rightarrow a^2=\frac{9}{4} \rightarrow \boxed{a=+\frac{3}{2}}$

$a=-\frac{3}{2}$ ×

$\cot\alpha=\frac{BD}{AB}=\frac{3}{\frac{3}{2}}=2$ (۲)



$AH=\sqrt{16-9}=\sqrt{7}$

$\tan(\pi-\alpha)=\frac{\sqrt{7}}{2}=-\tan\alpha \rightarrow \tan\alpha=\frac{-\sqrt{7}}{2}$ (۴)

$2\sin^2\alpha+\cos^2\alpha=\frac{4}{3}$

$\sin^2\alpha+\cos^2\alpha=1$

$\sin^2\alpha=\frac{1}{3} \rightarrow \cos^2\alpha=1-\sin^2\alpha=\frac{2}{3}$

$\tan^2\alpha=\frac{\sin^2\alpha}{\cos^2\alpha}=\frac{\frac{1}{3}}{\frac{2}{3}}=\boxed{\frac{1}{2}}$ (۵)

۶) $\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} =$

$\frac{(1-\cos^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(1-\sin^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{\cos^4\alpha + 2\cos^2\alpha + 1}{\cos^2\alpha + 1}$

$\frac{\sin^4\alpha + 2\sin^2\alpha + 1}{\sin^2\alpha + 1} = \cos^2\alpha + 1 - \sin^2\alpha \times 1 = \cos^2\alpha - \sin^2\alpha =$

$2\cos^2\alpha - 1 \quad \downarrow \quad 1 - 2\sin^2\alpha \quad \downarrow \quad \cos 2\alpha$

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۷) $\tan\alpha = \frac{4}{3}$

$\sin\left(\frac{9\pi}{2} + \alpha\right)\cos\left(\frac{7\pi}{2} - \alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right)$

$\cos\alpha(-\sin\alpha) + \cot\alpha = -\sin\alpha\cos\alpha + \cot\alpha = \frac{-12}{25} + \frac{3}{4} = \frac{-48+75}{100}$

$\frac{27}{100}$

$\sin\alpha = \frac{-4}{5}$

$\cos\alpha = \frac{-3}{5}$

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۸) $(2\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x)$

$\frac{3}{2} + \sqrt{2}(\sin x - \cos x)\left(-\frac{\sqrt{2}}{2}\right) = \frac{3}{2} - 1 = \frac{1}{2}$

$(\sin x - \cos x)^2 = 1 - 2\sin x\cos x = 1 - \sin 2x = \frac{1}{2} \rightarrow |\sin x - \cos x| = \frac{\sqrt{2}}{2}$

$\sin x < \cos x$

$\cos x - \sin x = \frac{\sqrt{2}}{2}$

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۹) $\tan\left(\frac{\alpha}{2}\right) = \frac{1}{4} \rightarrow \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{1}{4} \rightarrow 4 - 4\cos\alpha = 1 + \cos\alpha$

$3 = 5\cos\alpha \rightarrow \cos\alpha = \frac{3}{5}$

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{4}{3} - \frac{4}{5}}{\frac{1}{5}} = \frac{8}{15} \times 5 = \frac{8}{3}$

$\sin\alpha = \frac{4}{5}$

$\tan\alpha = \frac{4}{3}$

۹) $\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{8}{15}$ $\quad \cos\alpha = \frac{15}{17}$, $\sin\alpha = \frac{8}{17}$

موضوع:

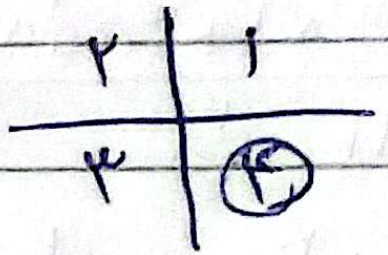
$$\frac{\tan\alpha + \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{-14}{105}$$

۱۰

$2\sin\alpha < \sin 2\alpha \rightarrow 0 < 2\sin\alpha\cos\alpha - 2\sin\alpha$

$0 < \underbrace{2\sin\alpha}_{(-)}(\underbrace{\cos\alpha - 1}_{(-)}) \rightarrow \sin\alpha < 0$

$0 < \frac{\cot\alpha}{\sin\alpha} \rightarrow 0 < \frac{\cos\alpha}{\sin^2\alpha} \rightarrow \cos\alpha > 0$



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