۱)

$$S_{ABC}^{\Delta} = \sqrt{۳} \times ۴ \times \frac{۱}{۲} \times \sin\alpha = ۴,۵ \rightarrow \sin\alpha = \frac{\sqrt{۳}}{۲} \begin{cases} \alpha = \frac{\pi}{۳} \\ \alpha = \frac{۲\pi}{۳} \end{cases} \rightarrow \frac{\frac{۲\pi}{۳}}{\frac{\pi}{۳}} = ۲$$

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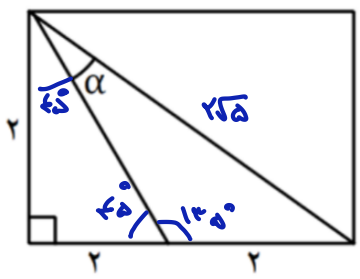
۲)



$$\frac{۲\sqrt{۵}}{\sin ۱۳۵^\circ} = \frac{۲}{\sin\alpha} \qquad \cot\alpha = ۳$$

$$\frac{۲\sqrt{۵}}{\frac{\sqrt{۲}}{۲}} = \frac{۲}{\sin\alpha} \rightarrow \sin\alpha = \frac{\sqrt{۲}}{۲\sqrt{۵}}$$

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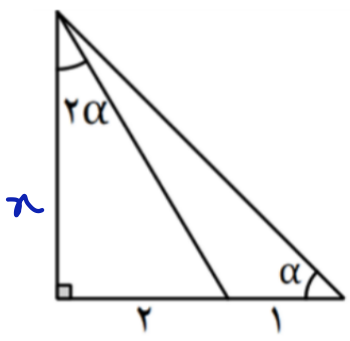
۳)



$$\cot ۲\alpha = \frac{x}{۲} = \frac{۱-\tan^۲\alpha}{۲\tan\alpha} \rightarrow \frac{۱-\frac{x^۲}{۹}}{\frac{۲x}{۳}} = \frac{۹-x^۲}{۶x} = \frac{x}{۲} \rightarrow ۹-x^۲ = ۳x^۲ \Rightarrow x = \frac{۳}{۲}$$

$$\tan\alpha = \frac{x}{۳}$$

$$\cot\alpha = \frac{۳}{x} \xrightarrow{x=\frac{۳}{۲}} \cot\alpha = ۲$$

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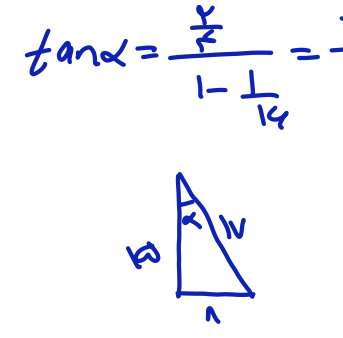
۴)



$$\tan(۱۳۰-\alpha) = -\tan\alpha = \frac{\sqrt{۷}}{۲} \Rightarrow \tan\alpha = -\frac{\sqrt{۷}}{۲}$$

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۵)

$$۲\sin^۲x + \cos^۲x = \frac{۶}{۴} \rightarrow \sin^۲x + \sin^۲x\cos^۲x = \frac{۳}{۴} \rightarrow \sin^۲x = \frac{۱}{۴} \rightarrow \cos^۲x = \frac{۳}{۴}$$

$$۱+\tan^۲x = \frac{۱}{\cos^۲x} = \frac{۴}{۳} \rightarrow \tan^۲x = \frac{۱}{۳}$$

---

۶)

$$\frac{\sin^۴\alpha + ۴(۱-\sin^۲\alpha)}{۲-\sin^۲\alpha} - \frac{\cos^۴\alpha + ۴(۱-\cos^۲\alpha)}{۲-\cos^۲\alpha} = \frac{\sin^۴\alpha - ۴\sin^۲\alpha + ۴}{۲-\sin^۲\alpha} - \frac{\cos^۴\alpha - ۴\cos^۲\alpha + ۴}{۲-\cos^۲\alpha}$$

$$= \frac{(\sin^۲\alpha - ۲)^۲}{۲-\sin^۲\alpha} - \frac{(\cos^۲\alpha - ۲)^۲}{۲-\cos^۲\alpha} = ۲-\sin^۲\alpha - ۲ + \cos^۲\alpha = \cos^۲\alpha - \sin^۲\alpha = \cos ۲\alpha$$

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۷)

$$\sin\left(\frac{۹\pi}{۲}+\alpha\right)\cos\left(\frac{۷\pi}{۲}-\alpha\right) - \tan\left(\alpha-\frac{۳\pi}{۲}\right) = \cos\alpha(-\sin\alpha) + \cot\alpha$$

$$-\frac{۳}{۵}\left(+\frac{۴}{۵}\right) + \frac{۳}{۴} = \frac{۲۷}{۱۰۰}$$

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۸)

$$۳\cos ۴x + \sqrt{۲}(\sin x - \cos x) = ۳\cos ۴x + \sqrt{۲}\left(\sqrt{۲}\sin\left(x-\frac{\pi}{۴}\right)\right) = ۳\cos\frac{\pi}{۳} - ۲\sin\frac{\pi}{۶} = \frac{۱}{۲}$$

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۹)

$$\tan\alpha = \frac{\frac{۲}{۴}}{۱-\frac{۱}{۱۴}} = \frac{\frac{۱}{۲}}{\frac{۱۳}{۱۴}} = \frac{۸}{۱۵}$$

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{۸}{۱۵} - \frac{۸}{۱۷}}{\frac{۸}{۱۷} - \frac{۱۵}{۱۷}} = \frac{\frac{۱۴}{۱۵\times۱۷}}{\frac{-۷}{۱۷}} = -\frac{۱۴}{۷\times۱۵} = -\frac{۱۴}{۱۰۵}$$

$\frac{\cot\alpha}{\sin\alpha} > 0 \longrightarrow \frac{\cos\alpha}{\sin^2\alpha} > 0 \xrightarrow{\sin^2\alpha > 0} \cos\alpha > 0$ (ناحیه اول یا چهارم) $\Longrightarrow$ ناحیه چهارم

$2\sin\alpha < \sin 2\alpha \longrightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \longrightarrow 0 < 2\sin\alpha(\underbrace{\cos\alpha - 1}_{-}) \longrightarrow \sin\alpha < 0$ (ناحیه سوم یا چهارم)

$0 < \cos\alpha < 1 \longrightarrow -1 < \cos\alpha - 1 < 0$