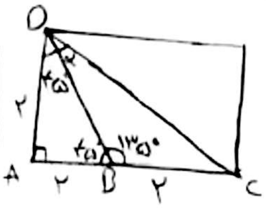
۱- $\frac{1}{2}\times 6\times \sqrt{3}\times \sin\alpha = 4{,}5 \to \sin\alpha = \frac{\sqrt{3}}{2}$ کمترین $\alpha = 60^\circ$ / بیشترین $\alpha = 120^\circ$ $\Rightarrow$ ۲ برابر

۲-



$OB=\sqrt{2^2+2^2}=2\sqrt{2}$, $DC=\sqrt{4+16}=2\sqrt{5}$

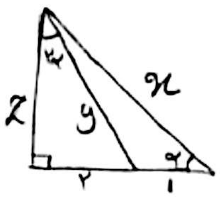
$S_{BOC}=S_{CDB} \Rightarrow \frac{1}{2}\times \underbrace{DC\times BD}_{2\sqrt{2}\times 2\sqrt{5}}\times \sin\alpha = \frac{1}{2}\times \underbrace{DB\times BC}_{2\sqrt{2}\times 2}\sin \overset{\frac{\sqrt{2}}{2}}{135}$

$\Downarrow$

$\sin\alpha = \frac{\sqrt{10}}{10}$

درنهایت: $1+\cot^2\alpha = \frac{1}{\underset{(\frac{\sqrt{10}}{10})^2}{\sin^2\alpha}} \Rightarrow \boxed{\cot\alpha = 3}$

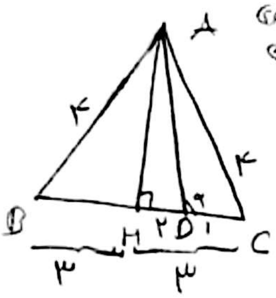
۳-



$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
$\sin 2\alpha = 2\sin\alpha\cos\alpha$
$\Longrightarrow \begin{cases} \frac{z}{y}=\left(\frac{3}{x}\right)^2-\left(\frac{z}{x}\right)^2 \Rightarrow \frac{9-z^2}{x^2}=\frac{x}{y} \\ \frac{2}{y}=2\times\frac{z}{x}\times\frac{3}{x}\Rightarrow x^2=3yz \end{cases}$ پس

$\frac{9-z^2}{3yz}=\frac{z}{y} \Rightarrow z=\frac{3}{2}$ درنهایت: $\cot\alpha = \frac{3}{\underset{\frac{3}{2}}{z}} = \boxed{2}$

۴-



مثلث متساوی الساقین $\to$ میانه = ارتفاع $\to BH=HC=3$

$AH=\sqrt{16-9}=\sqrt{7} \Rightarrow AD=\sqrt{(\underset{7}{\sqrt{7}})^2+2^2}=\sqrt{11}$

$4=\sqrt{(\sqrt{11})^2+1^2-2\sqrt{11}\cos\alpha} \to \cos\alpha = \frac{2\sqrt{11}}{11}$

$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \to \underset{\text{زاویه باز}}{\tan\alpha} = -\sqrt{\frac{7}{4}} = \frac{-\sqrt{7}}{2}$

۵- $\sin^2x+\underbrace{\sin^2x+\cos^2x}_{1}=\frac{4}{3} \to \sin^2x=\frac{1}{3}$, $\cos^2x=1-\frac{1}{3}=\frac{2}{3} \Rightarrow$

$\tan^2x=\frac{\sin^2x}{\cos^2x}=\frac{\frac{1}{3}}{\frac{2}{3}}=\frac{1}{2}$

۶-

$\sin^4\alpha = \underbrace{(1-\cos^2\alpha)^2}_{1-2\cos^2\alpha+\cos^4\alpha} \xrightarrow{+4\cos^2\alpha} \sin^4\alpha+4\cos^2\alpha = \underbrace{1+2\cos^2\alpha+\cos^4\alpha}_{(\cos^2\alpha+1)^2}$

$\cos^4\alpha = \underbrace{(1-\sin^2\alpha)^2}_{1-2\sin^2\alpha+\sin^4\alpha} \xrightarrow{+4\sin^2\alpha} \cos^4\alpha+4\sin^2\alpha = \underbrace{1+2\sin^2\alpha+\sin^4\alpha}_{(1+\sin^2\alpha)^2}$

$\Longrightarrow$

$\frac{(\cos^2\alpha+1)^2}{\cos^2\alpha+1}-\frac{(1+\sin^2\alpha)^2}{1+\sin^2\alpha} \Rightarrow \cos^2\alpha+1-1-\sin^2\alpha = \cos^2\alpha-\sin^2\alpha=\cos2\alpha$

$$\sin(\underbrace{4\pi}_{\text{۲ دور کامل}}+\frac{\pi}{2}+\alpha)\cos(\underbrace{\overbrace{3\pi}^{\text{یک دور و نیم}}+\frac{\pi}{2}}_{\frac{3\pi}{2}}-\alpha)-\tan(\alpha-\frac{3\pi}{2})=\cos\alpha(-\sin\alpha)+\cot\alpha$$ -۷

$$\underbrace{1+\tan^2\alpha}_{\frac{16}{9}}=\frac{1}{\cos^2\alpha}\Longrightarrow\cos\alpha=-\frac{3}{5}\ ,\ \sin\alpha=-\sqrt{1-\frac{9}{25}}=-\frac{4}{5}$$ ص)

درنهایت: $-\frac{3}{5}\times\left(-(-\frac{4}{5})\right)+\frac{3}{4}=\frac{27}{100}$

$$\sqrt{2}(\underbrace{\sin x-\cos x}_{\sqrt{2}\sin(x-\frac{\pi}{4})})=2\sin(\underbrace{\frac{\pi}{12}-\frac{\pi}{4}}_{-\frac{\pi}{6}})=-1$$ -۸

$$3\cos\underbrace{\frac{4\pi}{12}}_{\frac{\pi}{3}}=\frac{3}{2}$$

$$\Longrightarrow\frac{3}{2}-1=\boxed{\frac{1}{2}}$$ ص)

$$\tan\alpha=\frac{2\tan(\frac{\alpha}{2})}{1-\tan^2(\frac{\alpha}{2})}=\frac{2\times\frac{1}{4}}{1-\frac{1}{16}}=\frac{\frac{1}{2}}{\frac{15}{16}}=\frac{8}{15}\longrightarrow$$ -۹

$\to x=\sqrt{64+225}=17$ ص)

$$\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=-\frac{16}{105}$$

$$0<\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha}\longrightarrow\frac{\cos\alpha}{\underset{+}{\sin^2\alpha}}>0\longrightarrow\cos\alpha>0$$ -۱۰

$$2\sin\alpha<\sin2\alpha\Longrightarrow2\sin\alpha<2\sin\alpha\cos\alpha\longrightarrow0<\underbrace{2\sin\alpha}_{-}\underbrace{(\cos\alpha-1)}_{-}$$ ص)

$$\begin{cases}\sin\alpha<0\\ \cos\alpha>0\end{cases}\longrightarrow$$ ربع چهارم