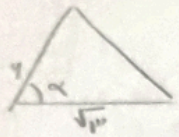
(۱)



$S_{\text{مثلث}} = \frac{۱}{۲} \times A \times B \sin \alpha \rightarrow \frac{۹}{۲} = ۴ \times \sqrt{۳} \times \frac{۱}{۲} \times \sin\alpha \rightarrow \sin\alpha = \frac{\sqrt{۳}}{۲}$ $\alpha = ۶۰$ $\alpha = ۱۲۰$

$\frac{۱۲۰}{۶۰} = ۲$

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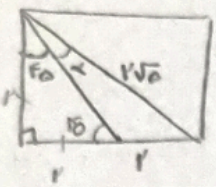
(۲)



$\cot(۴۵+\alpha) = \frac{۱}{۲}$ $\frac{۱}{۲} = \frac{\cot\alpha \cot ۴۵ - ۱}{۱ + \cot\alpha} \rightarrow ۱ + \cot\alpha = ۲\cot\alpha - ۲$

$۳ = \cot\alpha$

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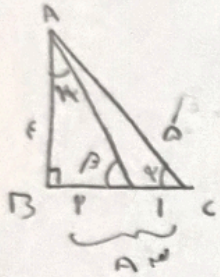
(۳)



$\rightarrow$ ۳، ۴، ۵ اعداد متناسب ← عکس قضیه تالس

$\cot\alpha = \frac{۳}{۴}$

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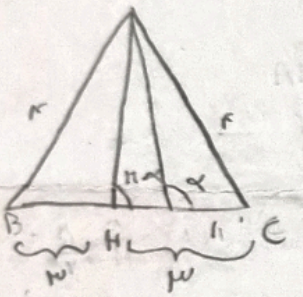
(۴)



$AC^۲ = AH^۲ + HC^۲ \rightarrow ۱۶ = AH^۲ + ۹ \rightarrow AH = \sqrt{۷}$

$\tan(۱۸۰-\alpha) = \frac{\sqrt{۷}}{۳} \rightarrow -\tan\alpha = \frac{\sqrt{۷}}{۳} \rightarrow \tan\alpha = -\frac{\sqrt{۷}}{۳}$

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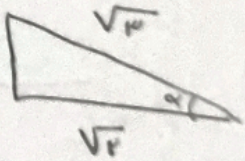
(۵)

$۲\sin^۲\alpha \cos^۲\alpha = \frac{۶}{۳} \rightarrow ۱ + \sin^۲\alpha = \frac{۴}{۳} \rightarrow \sin^۲\alpha = \frac{۱}{۳} \rightarrow \sin\alpha = \frac{۱}{\sqrt{۳}}$

$\tan^۲\alpha = \frac{۱}{\sqrt{۲}} = \frac{۱}{۲}$



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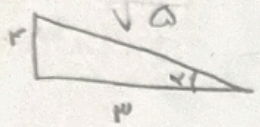
۷) $\sin\left(\frac{\pi}{2}+\alpha\right)\cos\left(\frac{3\pi}{2}-\alpha\right)-\tan\left(\alpha-\frac{3\pi}{2}\right) \rightarrow = \cos\alpha-\sin\alpha+\cot\alpha =$



$\frac{۳}{۵}-\frac{۴}{۵}+\frac{۳}{۴}=-\frac{۱۲}{۲۰}+\frac{۳}{۴}=\frac{-۴۸+۷۵}{۱۰۰}$

$=\frac{۲۷}{۱۰۰}$

$\frac{\pi}{۱۲}\times۱۵^\circ$

۸) $۳\cos۴x+\sqrt{۲}\underbrace{(\sin x-\cos x)}_{\sqrt{۲}\sin(۳۰)}=۳\cos۹۰+۲\sin۳۲۰\rightarrow ۳\times\frac{۱}{۲}+۲\times-\frac{۱}{۲}=\frac{۱}{۲}$

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۹) $\frac{\frac{\sin\alpha}{\cos\alpha}-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\sin\alpha-\sin\alpha\cos\alpha}{\sin\alpha\cos\alpha-\cos^۲\alpha}\Rightarrow\frac{\sin\alpha-\frac{۱}{۲}\sin۲\alpha}{\frac{۱}{۲}\sin۲\alpha-\cos^۲\alpha}$

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۱۰) $\frac{\cot۲}{\sin\alpha}>۰\rightarrow\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha}>۰\rightarrow\frac{\cos\alpha}{\sin^۲\alpha}>۰\rightarrow\cos\alpha>۰\rightarrow$ ربع اول و چهارم

$۲\sin\alpha<۲\sin\alpha\cos\alpha\rightarrow ۲\sin\alpha\cos\alpha-۲\sin\alpha>۰\rightarrow ۲\sin\alpha(\cos\alpha-۱)>۰$

$\sin\alpha<۰\rightarrow$ ربع سوم

(−) (+) (−)