نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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$S = \frac{1}{2} \times 4 \times \sqrt{3} \times \sin\alpha = \frac{9}{4} \rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$

$\rightarrow \alpha = \frac{\pi}{3}, \frac{2\pi}{3} \rightarrow$ (۲ یا ۳)

۲

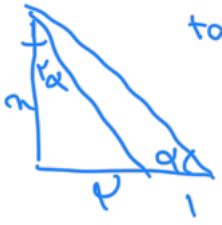
$\alpha + a + b = 90 \rightarrow \alpha = 90 - (a+b)$

$\tan a = \frac{2}{\alpha} = 1 \quad , \quad \tan b = \frac{1}{2}$

$\cot\alpha = \cot\left(\frac{\pi}{2} - (a+b)\right) = \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$

$= \frac{3/2}{1/2} = 3$ (۳)

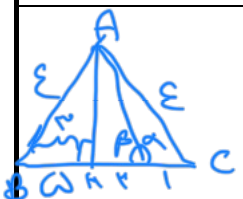
۳



$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \rightarrow \frac{2}{x} = \frac{2x}{1-x^2} = \frac{2x}{9-x^2} \rightarrow \frac{1}{x} = \frac{x}{9-x^2}$

$x^2 = 9 - x^2 \rightarrow x = \frac{3}{\sqrt{2}} \quad \cot\alpha = \frac{x}{2} = \frac{3/\sqrt{2}}{2} = \frac{3}{2\sqrt{2}}$ (۲)

۴



$\triangle AHC : 9 + AH^2 = 16 \rightarrow AH = \sqrt{7}$

$\tan\alpha = -\tan(\beta) = -\tan(180 - \alpha) = \frac{-\sqrt{7}}{3}$

۵

$\sin^2 x + \sin^2 2x + \cos^2 3x = \frac{6}{4} \rightarrow \sin^2 x = \frac{1}{4} \rightarrow \cos^2 2x = 1 - \frac{1}{4} = \frac{3}{4}$

$\tan^2 x = \frac{1/4}{3/4} = \frac{1}{3}$

۶

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} = \frac{\sin^4\alpha + 4 - 4\sin^2\alpha}{1+1-\sin^2\alpha} = \frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha} = 2-\sin^2\alpha$$

$$\frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{\cos^4\alpha + 4 - 4\cos^2\alpha}{1+1-\cos^2\alpha} = \frac{(2-\cos^2\alpha)^2}{2-\cos^2\alpha} = 2-\cos^2\alpha$$

$$2-\sin^2\alpha - 2 + \cos^2\alpha = \cos^2\alpha - \sin^2\alpha = \underline{\cos 2\alpha}$$

۷

$$+\cos\alpha \times -\sin\alpha - (-\cot\alpha) = -\frac{3}{5}\times\frac{4}{5} + \frac{3}{4} \quad (0/25)$$

$$1+\tan^2 = \frac{1}{\cos^2}$$

$$\rightarrow \cos\alpha = \frac{4}{5} \quad , \sin\alpha = \sqrt{1-\cos^2\alpha} = \frac{3}{5}$$

۸

$$\sin\alpha - \cos\alpha = \sqrt{2}\sin\left(\alpha - \frac{\pi}{4}\right)$$

$$\sqrt{2}(\sin\alpha - \cos\alpha) = \sqrt{2}\times\sqrt{2}\times\sin\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = 2\times\sin\left(\frac{\pi}{4}\right) = 1$$

$$2\cos\left(2\times\frac{\pi}{12}\right) = 2\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \rightarrow \frac{\sqrt{3}}{2} - 1 = \frac{-1}{2}$$

۹

$$\sin\alpha = \frac{2\times\tan\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{\frac{1}{2}}{\frac{17}{16}} = \frac{8}{17} \quad , \cos\alpha = \frac{1+\tan^2\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{\frac{16}{16}}{\frac{17}{16}} = \frac{15}{17}$$

$$\tan\alpha = \frac{\frac{8}{17}}{\frac{15}{17}} = \frac{8}{15} \rightarrow \frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{16}{255}}{\frac{-7}{17}} = \frac{-16}{105}$$

۱۰

$$\sin 2\alpha = 2\sin\alpha\cos\alpha \rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \xrightarrow{\sin\alpha > 0} \cos\alpha > \frac{1}{2} \quad \text{①}$$

$$\frac{\cot\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sin^2\alpha} > 0 \rightarrow \cos\alpha > 0 \quad \text{②}$$

①, ② → همه