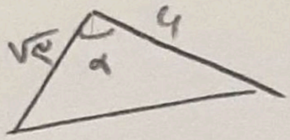
۱)



$S = \frac{۱}{۲} \times \sqrt{۳} \times ۶ \times \sin\alpha = \frac{۹}{۲}$

$\Rightarrow \sin\alpha = \sqrt{۳}/۲$ → Min α ۶۰° ; Max α ۳۰۰° → $\frac{۳۰۰}{۶۰} = ۵$]

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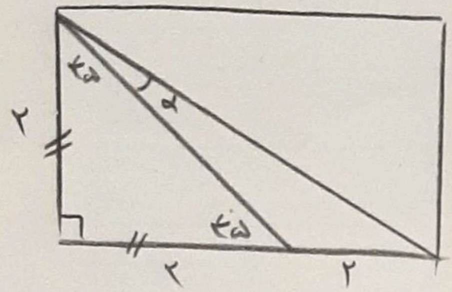
۲)



$\cot(\frac{\pi}{۴}+\alpha) = \frac{۱}{۲} = \frac{۱(\cot\alpha) - ۱}{۱ + \cot\alpha}$

$\Rightarrow \cot\alpha = ۳$]

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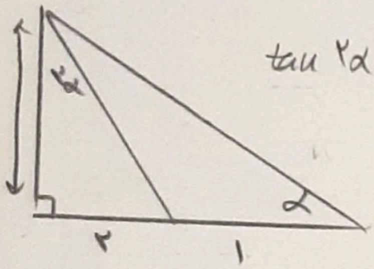
۳)



$\tan\alpha = \frac{۳}{h}$

$\tan ۲\alpha = \frac{h}{۲}$

$\tan ۲\alpha = \frac{۲\tan\alpha}{۱-\tan^۲\alpha} \to \frac{h}{۲} = \frac{\frac{۶}{h}}{۱-\frac{۹}{h^۲}}$; $۱-\frac{۹}{h^۲} \to \frac{h^۲-۹}{h^۲}$

$\frac{h}{۲} = \frac{۶h}{h^۲-۹} \to h^۳ - ۹h = ۱۲h$

$h^۳ - ۲۱h = ۰ \to h(h-\sqrt{۲۱})(h+\sqrt{۲۱}) \Rightarrow h = \sqrt{۲۱}$

غ ق ق $۰ =$ ، غ ق ق $-\sqrt{۲۱}$

$\to \tan\alpha = \frac{۳}{\sqrt{۲۱}} \Rightarrow \cot\alpha = \frac{\sqrt{۲۱}}{۳}$]

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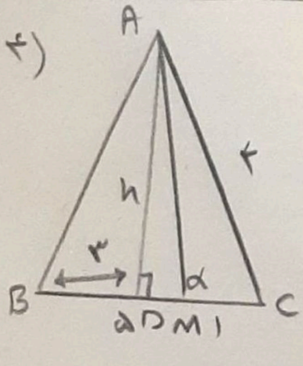
۴)



$AB^۲ = ۴^۲ = ۳^۲ + h^۲ \Rightarrow h = \sqrt{۷}$]

$B(۰,۰)$ $D(۵,۰)$

$C(۶,۰)$

$A(۳,\sqrt{۷})$

شیب AD : $-\frac{\sqrt{۷}}{۲} \to |-\frac{\sqrt{۷}}{۲}| = \sqrt{۷}/۲$]

آ لیدا دبی

۵) $2\sin^2 x + \cos^2 x = \frac{4}{3} \to \tan^2 = ?$

$\div \cos^2$: $2\tan^2 = \frac{4}{3} - \frac{3}{3} \to \tan^2 = \frac{1}{4}$

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۶) $\dfrac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \dfrac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha}$

صورت کسر اول : $\dfrac{\sin^4\alpha + 4(1-\sin^2\alpha)}{1+(1-\sin^2\alpha)} = \dfrac{\sin^4\alpha - 4\sin^2\alpha + 4}{2-\sin^2\alpha} = \dfrac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha}$

به طور مشابه، کسر دوم : $\dfrac{\cos^4\alpha - 4\cos^2\alpha + 4}{2-\cos^2\alpha} = 2-\cos^2\alpha$

$\to 2-\sin^2\alpha - 2 + \cos^2\alpha = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

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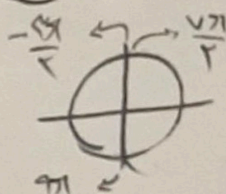
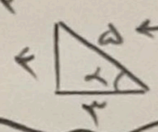
۷) $\sin(\frac{9\pi}{2}+\alpha)\times\cos(\frac{7\pi}{2}-\alpha) - \tan(\alpha - \frac{3\pi}{2})$



$= \cos\alpha \times \sin\alpha + \cot\alpha = -\frac{3}{5}\times\frac{4}{5} + \frac{3}{4}$

$= \dfrac{-48+75}{100} = 0.27$

نسبت‌های مثلثاتی



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۸) $3\cos\frac{\pi}{6} + \sqrt{2}\sin\frac{\pi}{12} - \sqrt{2}\cos\frac{\pi}{12}$

$\frac{\pi}{12} \leftrightarrow 15^\circ$

$\sin 15^\circ = \sin(45-30) = \sin 45\cos 30 - \cos 45 \sin 30 = \frac{\sqrt6-\sqrt2}{4}$ تفاضل $\to -\frac{\sqrt2}{2}$

$\cos 15^\circ = \cos(45-30) = \cos 45\cos 30 - \sin 45 \sin 30 = \frac{\sqrt6+\sqrt2}{4}$

$\to 3\times\frac{\sqrt3}{2} + \sqrt2 \times \frac{-\sqrt2}{2} = \frac{3\sqrt3}{2} - \frac{2}{2} \to \frac{3\sqrt3-2}{2}$

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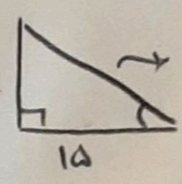
۹) $\tan(\alpha) = \dfrac{2\tan(\alpha/2)}{1-\tan^2(\frac{\alpha}{2})} \to \tan\alpha = \frac{8}{15}$



$\to \sqrt{225+64} = 17$

$\dfrac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \dfrac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{256}{105}$