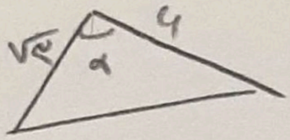
۱۴

1)



$S = \frac{1}{2} \times \sqrt{3} \times 4 \times \sin\alpha = \frac{9}{2}$

$\Rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$ → Min α = 60° ; Max α = ~~30°~~ 120 → $\frac{120}{60} = 2$

۱,۵

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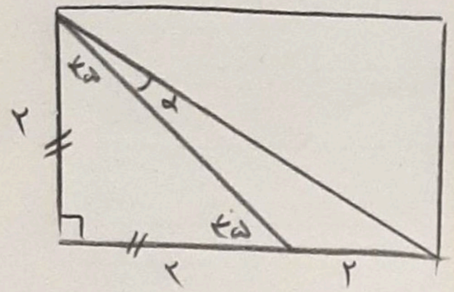
2)



$\cot(\frac{\pi}{4} + \alpha) = \frac{1}{2} = \frac{1(\cot\alpha) - 1}{1 + \cot\alpha}$

$\Rightarrow \cot\alpha = 3$

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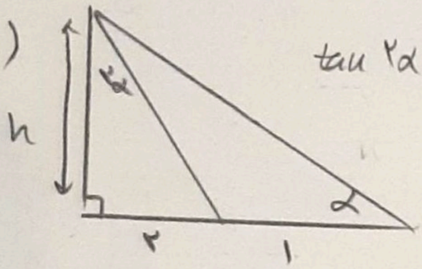
3)



$\tan\alpha = \frac{3}{h}$

$\tan 2\alpha = \frac{h}{2}$

$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha} \rightarrow \frac{h}{2} = \frac{\frac{6}{h}}{1 - \frac{9}{h^2}}$, $1 - \frac{9}{h^2} \rightarrow \frac{h^2 - 9}{h^2}$

$\frac{h}{2} = \frac{6h}{h^2 - 9} \rightarrow h^3 - 9h = 12h$

$h^3 - 21h = 0 \rightarrow h(h - \sqrt{21})(h + \sqrt{21}) \Rightarrow h = \sqrt{21}$

$\rightarrow \tan\alpha = \frac{3}{\sqrt{21}} \Rightarrow \cot\alpha = \frac{\sqrt{21}}{3}$

۰,۵

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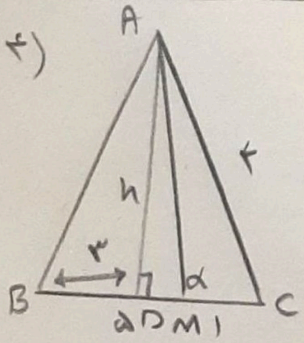
4)



$AB^2 = 4^2 = 3^2 + h^2 \rightarrow h = \sqrt{7}$

B(0, 0)   M(5, 0)

C(6, 0)

A(3, √7)

شیب AD : $-\frac{\sqrt{7}}{2} \rightarrow \left|-\frac{\sqrt{7}}{2}\right| = \frac{\sqrt{7}}{2}$

۱,۷۵

۵) $2\sin^2x+\cos^2x=\frac{4}{3} \to \tan^2=?$

$\div\cos^2$: $2\tan^2=\frac{4}{3}-\frac{3}{3} \to \tan^2=\frac{1}{4}$]

آ لکادری ۵

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۶) $\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}$

کسر اول: $\frac{\sin^4\alpha+4(1-\sin^2\alpha)}{1+(1-\sin^2\alpha)}=\frac{\sin^4\alpha-4\sin^2\alpha+4}{2-\sin^2\alpha}=\frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha}$

به طور مشابه، کسر دوم: $\frac{\cos^4\alpha-4\cos^2\alpha+4}{2-\cos^2\alpha}=2-\cos^2\alpha$

$\to 2-\sin^2\alpha-2+\cos^2\alpha=\cos^2\alpha-\sin^2\alpha=\cos2\alpha$]

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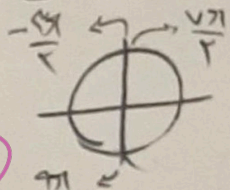
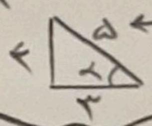
۷) $\sin(\frac{9\pi}{2}+\alpha)\times\cos(\frac{7\pi}{2}-\alpha)-\tan(\alpha-\frac{3\pi}{2})$



$=-\cos\alpha\times\sin\alpha+\cot\alpha=-\frac{3}{5}\times\frac{4}{5}+\frac{3}{4}$

$=\frac{-48+75}{100}=0/27$

فیثاغورس



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۸) $3\cos\frac{\pi}{6}+\sqrt{2}\sin\frac{\pi}{12}-\sqrt{2}\cos\frac{\pi}{12}$

$\frac{\pi}{12} \leftrightarrow 15°$

$\sin15°=\sin(45-30)=\sin45\cos30-\cos45\sin30=\frac{\sqrt6-\sqrt2}{4}$ تفاضل $\to -\sqrt2/2$

$\cos15°=\cos(45-30)=\cos45\cos30-\sin45\sin30=\frac{\sqrt6+\sqrt2}{4}$

$\to 3\times\frac{\sqrt3}{2}+\sqrt2\times\frac{-\sqrt2}{2}=\frac{3\sqrt3}{2}-\frac{2}{2}\to\frac{3\sqrt3-2}{2}$]

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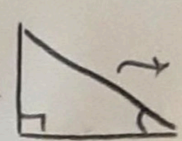
۹) $\tan(\alpha)=\frac{2\tan(\alpha/2)}{1-\tan^2(\frac{\alpha}{2})}\to\tan\alpha=\frac{8}{15}$



$\to\sqrt{225+64}=17$

$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\frac{16}{105}$ $\quad\frac{-16}{105}$

۱/۷۵

۳) $\tan B = \frac{AD}{AB} \to \tan 2\alpha = \frac{2}{x}$ $\quad \tan C = \frac{AB}{AC} \to \tan\alpha = \frac{x}{3}$

$\to \tan 2\alpha \to \frac{2}{x} = \frac{2 \times \frac{x}{3}}{1 - \frac{x^2}{9}} \to x = \frac{3}{2}$ $\quad \tan\alpha = \frac{1}{2}$, $\cot\alpha = 2$

۵) $\sin^2 x + \underbrace{\sin^2\alpha + \cos^2 x}_{1} = \frac{5}{3} \to \sin^2 x = \frac{1}{3}$, $\cos^2 x = \frac{2}{3}$

$\tan^2 x = \frac{1}{2}$

۱) $\frac{3}{2} + \sqrt{2}\underbrace{\left(\sin\frac{\pi}{12} + \cos\frac{\pi}{12}\right)}_{A}$ $\quad A^2 = 1 - \sin\frac{\pi}{4} = 1 - \frac{1}{2} = \frac{1}{2} \xrightarrow[A<0]{\sqrt{\ }} A = \frac{-1}{\sqrt{2}}$

$\frac{3}{2} + \sqrt{2} \times \frac{-1}{\sqrt{2}} = \frac{1}{2}$