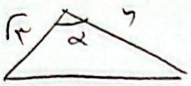
تکلیف شماره ۲۷ العیب جلالی

۱-

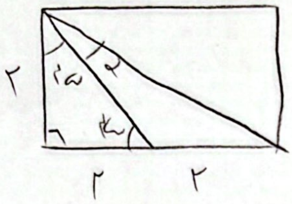
$S=\frac{1}{2}\times 6\times\sqrt{3}\times\sin\alpha = 4.5$



$\sin\alpha = \frac{1.5}{\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\left.\begin{array}{l}\alpha_{min}=60\\ \alpha_{max}=120\end{array}\right\} \frac{120}{60}=2$

۲-

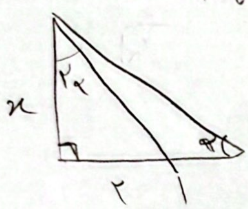


$\tan(\alpha+45) = \frac{\tan\alpha+\tan 45}{1-\tan\alpha\tan 45} = 2 \qquad \frac{\tan\alpha+1}{1-\tan\alpha} = 2$

$2-2\tan\alpha = \tan\alpha+1 \qquad 3\tan\alpha = 1 \qquad \tan\alpha = \frac{1}{3} \qquad \cot\alpha = 3$

۳-

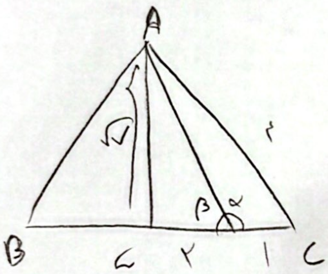
$\left.\begin{array}{l}\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2}{x}\\ \tan\alpha = \frac{x}{3}\end{array}\right\} \rightarrow \frac{\frac{2x}{3}}{1-\frac{x^2}{9}} = \frac{2x}{3-\frac{x^2}{3}} = \frac{6x}{9-x^2} = \frac{2}{x}$



$3x^2 = 9-x^2 \qquad 4x^2 = 9 \qquad x=\frac{3}{2}$

$\cot\alpha = \frac{3}{\frac{3}{2}} = 2$

۴-



$\tan\alpha = -\tan\beta = -\frac{\sqrt{7}}{2}$

۵-

$\sin^2x + (\sin^2x+\cos^2x) = \frac{4}{3} \qquad \sin^2x = \frac{1}{3} \qquad \cos^2x = \frac{2}{3} \qquad \tan^2x = \frac{\sin^2x}{\cos^2x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

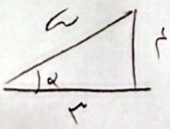
۶-

$\frac{\sin^4\alpha+2\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha+2\sin^2\alpha}{1+\sin^2\alpha} = \frac{\sin^4\alpha-4\sin^2\alpha+4}{2-\sin^2\alpha} - \frac{\cos^4\alpha-4\cos^2\alpha+4}{2-\cos^2\alpha}$

$\frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha} - \frac{(2-\cos^2\alpha)^2}{1-\cos^2\alpha} = 2-\sin^2\alpha-2+\cos^2\alpha = \cos^2\alpha-\sin^2\alpha = \cos 2\alpha$

۷-

$\cos\alpha\times(-\sin\alpha)+\cot\alpha = -\sin\alpha\cos\alpha+\cot\alpha = \frac{-12}{25}+\frac{3}{4} = \frac{-48+75}{100} = \frac{27}{100}$



$\cot\alpha = \frac{3}{4}$

$\sin\alpha = -\frac{4}{5}$

$\cos\alpha = -\frac{3}{5}$

۸-

$۳\cos ۲x + ۴\sin(x - \frac{۳\pi}{۴}) = ۳\cos\frac{\pi}{۳} - ۴\sin\frac{۳\pi}{۴} = \frac{۳}{۲} - \sqrt{۳}$

$\frac{\pi}{۱۲} - \frac{۳\pi}{۴} = -\frac{۸\pi}{۱۲} = -\frac{۲\pi}{۳}$

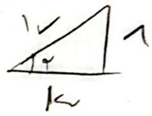
۹-

$\tan^۲(\frac{\alpha}{۲}) = \frac{۱-\cos\alpha}{۱+\cos\alpha} = \frac{۱}{۱۶}$ $\quad ۱۶ - ۱۶\cos\alpha = ۱ + \cos\alpha$ $\quad ۱۷\cos\alpha = ۱۵$

$\cos\alpha = \frac{۱۵}{۱۷}$

$\tan\alpha = \frac{۸}{۱۵}$

$\sin\alpha = \frac{۸}{۱۷}$



$$\frac{\frac{۸}{۱۵} - \frac{۸}{۱۷}}{\frac{۸}{۱۷} - \frac{۱۵}{۱۷}} = \frac{\frac{۸\times۱۷}{۱۵} - ۸}{-۷} = \frac{\frac{۱۶}{۱۵}}{-۷} = \frac{-۱۶}{۱۰۵}$$

۱۰-

$۲\sin\alpha < \sin۲\alpha$ $\quad ۴\sin\alpha\cos\alpha > ۴\sin\alpha$ $\quad \sin\alpha\cos\alpha > \sin\alpha$

$\sin\alpha > ۰ \rightarrow \cos\alpha > ۱$ ✗

$\sin\alpha < ۰ \rightarrow \cos\alpha < ۱$ ✓

$\frac{\cos\alpha}{\sin^۲\alpha} \geq ۰$ $\quad \cos\alpha > ۰$

پس $\sin\alpha < ۰$ و $\cos\alpha > ۰$ ← ربع چهارم