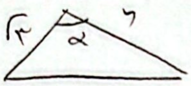
تکلیف شماره ۲۷ الهه جلالی

۱۸٫۵

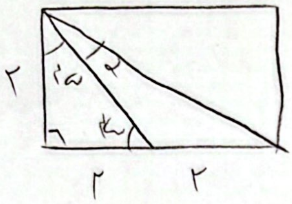
۱-

$S = \frac{1}{2} \times 4 \times \sqrt{3} \times \sin\alpha = 1.5 \qquad \sin\alpha = \frac{1.5}{\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$



$\left.\begin{array}{l}\alpha_{min} = 60 \\ \alpha_{max} = 120\end{array}\right\} \quad \frac{120}{60} = 2$

۲-

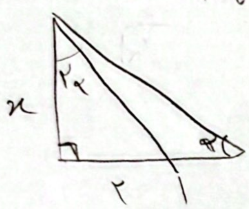


$\tan(\alpha + 2\alpha) = \frac{\tan\alpha + \tan 2\alpha}{1 - \tan\alpha \tan 2\alpha} = 2 \qquad \frac{\tan\alpha + 1}{1 - \tan\alpha} = 2$

$2 - 2\tan\alpha = \tan\alpha + 1 \qquad 3\tan\alpha = 1 \qquad \tan\alpha = \frac{1}{3} \qquad \cot\alpha = 3$

۳-

$\left.\begin{array}{l}\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha} = \frac{2}{x} \\ \tan\alpha = \frac{x}{3}\end{array}\right\} \to \frac{\frac{2x}{3}}{1 - \frac{x^2}{9}} = \frac{2x}{3 - \frac{x^2}{3}} = \frac{6x}{9 - x^2} = \frac{2}{x}$

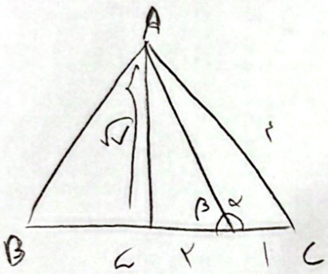


$3x^2 = 9 - x^2 \qquad 4x^2 = 9 \qquad x = \frac{3}{2}$

$\cot\alpha = \frac{3}{\frac{3}{2}} = 2$

۴-

$\tan\alpha = -\tan\beta = -\frac{\sqrt{7}}{2}$



۵-

$\sin^2 x + (\sin^2 x + \cos^2 x) = \frac{4}{3} \qquad \sin^2 x = \frac{1}{3} \qquad \cos^2 x = \frac{2}{3} \qquad \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

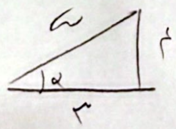
۶-

$\frac{\sin^2\alpha + 2\cos^2\alpha}{1 + \cos^2\alpha} - \frac{\cos^2\alpha + 2\sin^2\alpha}{1 + \sin^2\alpha} = \frac{\sin^2\alpha - 2\sin^2\alpha + 2}{2 - \sin^2\alpha} - \frac{\cos^2\alpha - 2\cos^2\alpha + 2}{2 - \cos^2\alpha}$

$\frac{(2 - \sin^2\alpha)^2}{2 - \sin^2\alpha} - \frac{(2 - \cos^2\alpha)^2}{1 - \cos^2\alpha} = 2 - \sin^2\alpha - 2 + \cos^2\alpha = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

۷-

$\cos\alpha \times (-\sin\alpha) + \cot\alpha = -\sin\alpha\cos\alpha + \cot\alpha = \frac{-12}{25} + \frac{3}{4} = \frac{-48 + 75}{100} = \frac{27}{100}$



$\cot\alpha = \frac{3}{4}$

$\sin\alpha = -\frac{4}{5}$

$\cos\alpha = -\frac{3}{5}$

−۸ (۰/۵)

$۳\cos ۲x + ۲\sin\left(x - \frac{۳\pi}{۴}\right) = ۳\cos\frac{\pi}{۳} - ۲\sin\frac{۳\pi}{۴} = \frac{۳}{۲} - \sqrt{۲}$

$\frac{\pi}{۱۲} - \frac{۳\pi}{۴} = \frac{-۸\pi}{۱۲} = \frac{-۲\pi}{۳}$

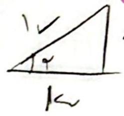
−۹

$\tan^۲\left(\frac{\alpha}{۲}\right) = \frac{۱-\cos\alpha}{۱+\cos\alpha} = \frac{۱}{۱۶}$ $\quad ۱۶ - ۱۶\cos\alpha = ۱ + \cos\alpha$ $\quad ۱۷\cos\alpha = ۱۵$

$\cos\alpha = \frac{۱۵}{۱۷}$

$\tan\alpha = \frac{۸}{۱۵}$ (۱)

$\sin\alpha = \frac{۸}{۱۷}$



$\dfrac{\frac{۸}{۱۵} - \frac{۸}{۱۷}}{\frac{۸}{۱۷} - \frac{۱۵}{۱۷}} = \dfrac{\frac{۸\times۱۷}{۱۵} - ۸}{-۷} = \dfrac{\frac{۱۶}{۱۵}}{-۷} = \frac{-۱۶}{۱۰۵}$

−۱۰

$۲\sin\alpha < \sin ۲\alpha$ $\quad ۲\sin\alpha\cos\alpha > ۲\sin\alpha$ $\quad \sin\alpha\cos\alpha > \sin\alpha$

$\sin\alpha > ۰ \rightarrow \cos\alpha > ۱$ ✗

$\sin\alpha < ۰ \rightarrow \cos\alpha < ۱$ ✓ (۱)

$\frac{\cos\alpha}{\sin^۲\alpha} \geq ۰ \quad \cos\alpha > ۰$

پس $\sin\alpha < ۰$ و $\cos\alpha > ۰$ ← ربع چهارم

۸) $\frac{۳}{۲} + \sqrt{۲}\underbrace{\left(\sin\frac{\pi}{۱۲} + \cos\frac{\pi}{۱۲}\right)}_{A}$

$A^۲ = ۱ - \sin\frac{\pi}{۶} = ۱ - \frac{۱}{۲} = \frac{۱}{۲} \xrightarrow[A<۰]{\sqrt{}} A = \frac{-۱}{\sqrt{۲}}$

$\frac{۳}{۲} + \sqrt{۲} \times \frac{-۱}{\sqrt{۲}} = \frac{۱}{۲}$