1

$$f(x)=3^{Ax+B}$$

$$f(1)=3^{A+B}=1 \Rightarrow A+B=0$$

$$f(3)=3^{3A+B}=3^2 \Rightarrow 3A+B=2$$

$$\ominus \rightarrow 2A=2 \rightarrow A=1,\quad B=-1$$

$$\boxed{f(0)=3^{-1}=\frac{1}{3}}$$

2

$$\log_2(4^x+15)=x+3 \rightarrow 4^x+15=2^{x+3} \xrightarrow[t=2^x]{} t^2-8t+15=0 \rightarrow (t-3)(t-5)=0$$

$$t=3 \text{ یا } t=5$$

$$2^x=3 \rightarrow x=\log_2 3,\quad 2^x=5 \rightarrow x=\log_2 5 \Rightarrow \boxed{\log_2 5+\log_2 3=\log_2 15}$$

3

$$(\log_{21}3)^2+\underbrace{\log_{21}147}_{\log_{21}3^2\times7^2}\ \underbrace{\log_{21}1323}_{\log_{21}7^2\times3^3}=(\log_{21}3)^2+\log_{21}(3\times7^2)\log_{21}(3\times7^2\times9)$$

$$=(\log_{21}3)^2+(\log_{21}3+2\log_{21}7)(3\log_{21}3+2\log_{21}7)=4(\log_{21}3+\log_{21}7)^2$$

$$\boxed{=4}$$

4

$$\log(\overset{(1-x)^2}{x^2-2x+1})+3\log(1-x)=5$$

$$2\log(1-x)+3\log(1-x)=5 \rightarrow 5\log(1-x)=5 \rightarrow \log(1-x)=1$$

$$\rightarrow 1-x=10 \Rightarrow x=-9$$

$$\boxed{\log_3 9=2}$$

۵

$$\log_{۲}(x^{۲}+۲x+۴)+\log_{۲}(x-۲)=۳$$

ادغام

$$\log_{۲}(x^{۳}-۸)=۳ \longrightarrow x^{۳}-۸=۸ \longrightarrow x=\sqrt[۳]{۱۶} \qquad \log_{\sqrt[۳]{۲}}\sqrt[۳]{۱۶}=۴$$

۶

$$\log(۲-x)-\log\frac{۱}{(x-۲)^{۲}}=۳$$

$$\log(۲-x)+۲\log(۲-x)=۳ \longrightarrow ۳\log(۲-x)=۳ \longrightarrow \log(۲-x)=۱$$

$$\longrightarrow ۲-x=۱۰ \longrightarrow x=-۸$$

$$\log_{\sqrt{۲}}۸=\boxed{۶}$$

۷

$$۳^{x^{۲}-۲}=۸۱^{x} \longrightarrow x^{۲}-۲=۴x \longrightarrow x^{۲}-۴x-۲=۰ \longrightarrow x=\frac{۴\pm\sqrt{۲۴}}{۲}=۲\pm\sqrt{۶}$$

غ‌ق‌ق

$$\log_{۶}(x-۲) \longrightarrow \log_{۶}\sqrt{۶}=\boxed{\frac{۱}{۲}}$$

۸

$$\log_{۳}۲=\frac{۵}{۸}$$

$$\log_{۱۸}۸=\frac{\log_{۳}۸}{\log_{۳}۱۸}=\frac{۳\log_{۳}۲}{۲+\log_{۳}۲}=\frac{۳\times\frac{۵}{۸}}{۲+\frac{۵}{۸}}=\frac{\frac{۱۵}{۸}}{\frac{۲۱}{۸}}=\boxed{\frac{۵}{۷}}$$

۹

$$\log_{۴}۳=۰٫۸$$

$$\log_{۱۲}۶=\frac{\log_{۴}۶}{\log_{۴}۱۲}=\frac{\log_{۴}۳+\log_{۴}۲}{\log_{۴}۳+\log_{۴}۴}=\frac{۰٫۸+\frac{۱}{۲}}{۰٫۸+۱}=\frac{۱٫۳}{۱٫۸}=\boxed{\frac{۱۳}{۱۸}}$$

10

$$(a\log 2)x^2 + ax + b\log 2 = 0 \xrightarrow{x=-1} a\log 2 - a + b\log 2 = 0$$

$$a\log 2 + b\log 2 = a$$

$$\log 2 \times (a+b) = a \xrightarrow[a\neq 0]{\div a} \left(1+\frac{b}{a}\right)\log 2 = 1$$

$$\log_2 10 = 1 + \frac{b}{a} = \frac{1}{\log 2}$$

$$\frac{b}{a} = \log_2 10 - 1 = \log_2 10 - \log_2 2 = \log_2 5$$

$$\to \sqrt{2}^{\frac{b}{a}} = 2^{\frac{b}{2a}} = 2^{\frac{1}{2}\log_2 5} = 2^{\log_2 \sqrt{5}} = \boxed{\sqrt{5}}$$