لعیا محسنی - تکلیف شماره ۲۴

۱-

$x=1 \to 1=3^{A+B}$

$x=3 \to 9=3^{3A+B}$

$A+B=0$

$3A+B=2$

$2A=2 \to A=1 \ ,\ B=-1$

$3^{-1}=\frac{1}{3}$

۲-

$\log_2^{4^x+15} = x+3 \Rightarrow 4^x+15 = 2^{x+3} \Rightarrow 2^{2x}-2^{x+3}+15=0$

$(2^x)^2 - 8\times 2^x + 15 = 0 \to 2^x = t \to t^2-8t+15=0 \to (t-3)(t-5)=0$

$t=3 \to 2^x=3 \to x=\log_2^3$

$t=5 \to 2^x=5 \to x=\log_2^5$

$\log_2^3+\log_2^5=\log_2^{15}$

۳-

$(\log_{21}^3)^2 + (\log_{21}^7+\log_{21}^{21})(\log_{21}^3+2\log_{21}^{21}) \longrightarrow \log_{21}^7 = 1-\log_{21}^3$

$(\log_{21}^3)^2 + (2-\log_{21}^3)(2+\log_{21}^3) \longrightarrow (\log_{21}^3)^2+4-(\log_{21}^3)^2=4$

۴-

$\log^{(1-x)^2}+3\log^{1-x}=5 \qquad 5\log(1-x)=5 \to \log(1-x)=1 \to 1-x=10,\ x=-9$

$\log_3^9=2$

۵-

$\log_2^{x^2+2x+4}+\log_2^{x-2} \to \log_2^{x^3-8}=3 \qquad \log_{\sqrt[3]{2}}^{\sqrt[3]{16}} = \frac{\frac{4}{3}}{\frac{1}{3}}=4$

$x^3-8=8 \to x^3=16 \to x=\sqrt[3]{16}$

۶-

$\log^{2-x}-\log^{(2-x)^{-2}}=3 \to 3\log^{2-x}=3 \to \log^{2-x}=1 \qquad 2-x=10 \to x=-8$

$\log_{\sqrt{2}}^8=\frac{3}{\frac{1}{2}}=6$

۷-

$3^{x^2-2}=3^{4x} \qquad x^2-4x-2=0 \to (x-2)^2=6 \to x-2=\sqrt{6} \to x=2+\sqrt{6}$

$x-2=-\sqrt{6} \to x=2-\sqrt{6}$ غیرقابل قبول

$\log_6^{2+\sqrt{6}-2}=\log_6^{\sqrt{6}}=\frac{1}{2}$

۸-

$\log_{18}^8=\frac{\log_2^8}{\log_2^{18}}=\frac{3}{\log_2^9+\log_2^2}=\frac{3}{1+2\log_2^3}=\frac{3}{1+2\times\frac{8}{5}}=\frac{3}{\frac{21}{5}}=\frac{5}{7}$

۹-

$\log_{12}^6=\frac{\log_2^6}{\log_2^{12}}=\frac{\log_2^3+1}{\log_2^3+2}=\frac{1.6+1}{1.6+2}=\frac{2.6}{3.6}=\frac{13}{18}$

$\frac{1}{2}\log_2^3=\frac{8}{10}$

$\log_2^3=1.6$

-10

$x = -1 \rightarrow a\log 2 - a + b\log 2 = 0 \rightarrow \log 2 = y \rightarrow ay - a + by = 0$

$\xrightarrow{\div a} y - 1 + \frac{b}{a}y = 0 \rightarrow y - \frac{b}{a}y = 1 \rightarrow 1 - \frac{b}{a} = \frac{1}{\log 2}$

$1 - \frac{b}{a} = \log_2 10 \rightarrow 1 - \frac{b}{a} = 1 + \log_2 5 \rightarrow \frac{b}{a} = -\log_2 5$

$\sqrt{2}^{\,-\log_2 5} = 5^{-\frac{1}{2}} = \sqrt{5}$