۱- $x=1 \longrightarrow 1=3^{A+B} \Rightarrow A+B=0$

$x=3 \longrightarrow 9=3^{3A+B} \Rightarrow 3A+B=2 \xrightarrow{A=-B} B=-1 \, , A=1 \Rightarrow f(x)=3^{x-1} \xrightarrow[x=0]{} f(0)=\frac{1}{3}$

۲- $\log_2^{(2^x+10)} = x+3 \Rightarrow 2^x+10=2^{x+3} \longrightarrow 2^{2x}-8\times 2^x+10=0 \xrightarrow{2^x=t} t^2-8t+10=0$

$(t-2)(t-5)=0 \Rightarrow 2 \, , 5$

$\begin{cases} 2^{x_1}=2 \longrightarrow \log_2^2=x_1 \\ 2^{x_2}=5 \longrightarrow \log_2^5=x_2 \end{cases} \oplus \Rightarrow \log_2^2+\log_2^5=\log_2^{10}$

۳- $\log_{21}^{147} = \log_{21}^{(21)^2} \Rightarrow \log_{21}^{21^2}-\log_{21}^3=2-\log_{21}^3$

$\log_{21}^{1323} = \log_{21}^{21^2\times 3} = 2+\log_{21}^3$

$\xrightarrow{\log_{21}^3=t} t^2+(2-t)(2+t)=4$

$4-t^2$

۴- $\log(x-1)^2-\underbrace{3\log(x-1)}_{\log(x-1)^3}=5 \Rightarrow -\log_{10}^{(x-1)^5}=5 \longrightarrow -(x-1)^5=10^5 \longrightarrow x=-9$

$\log_3^{-x}=\log_3^9=2$

۵- $\log_2^{(x^3+2x+4)(x-2)}=3 \Rightarrow 2^3=x^3-8 \longrightarrow x^3=16 \longrightarrow x=\sqrt[3]{16}=2^{\frac{4}{3}}$

$\log_{2^{\frac{1}{3}}}^{2^{\frac{4}{3}}}=\frac{\frac{4}{3}}{\frac{1}{3}}=4$

۶- $\log^{(2-x)}-\log\frac{1}{(2-x)^2}=3 \Rightarrow 3\log(2-x)=3 \longrightarrow \log(2-x)=1 \longrightarrow x=-8$

$\frac{1}{(2-x)^2}=(2-x)^{-2}$

$\log_{2^{\frac{1}{3}}}^{2^3}=6$

۷- $3^{x^2-2}=3^{4x} \longrightarrow x^2-4x-2=0$

$\Delta=16+8=24 \longrightarrow x=\frac{4\pm\sqrt{24}}{2}=2\pm\sqrt{6}$ (غ ق ق)

$\Rightarrow \log_6^{\sqrt{6}}=\frac{1}{2}$

۸- $\log_{12}^8=\frac{\log_3^8}{\log_3^{12}}=\frac{3\log_3^2}{\log_3^{3\times 4}}=\frac{3\log_3^2}{\log_3^3+2}=\frac{\frac{15}{8}}{\frac{21}{8}}=\frac{15}{21}=\frac{5}{7}$

۹-

$$\frac{\log_4^6}{\log_4^{12}} = \frac{\log_4^2 + \log_4^3}{1 + \log_4^3} = \frac{\frac{1}{2} + 0/8}{1 + 0/8} = \frac{1/3}{1/8} = \frac{13}{18}$$

۱۰-

چون یکی از ریشه‌ها ۱- هست

بس طبق $a+c=b$ →

$$a = \underbrace{a(\log 2) + b\log 2}_{\log 2 \times (a+b)} \longrightarrow \log 2 = \frac{a}{a+b} \xrightarrow{\text{معکوس}} \log_2^{10} = \frac{a+b}{a} = 1 + \frac{b}{a}$$

$$\log_2^{10} - 1 = \frac{b}{a} \Longrightarrow (\sqrt{2})^{\frac{b}{a}} = 2^{\frac{1}{2} \times \log_2^5}$$

$\log_2^{10} - \log_2^2 = \log_2^5$

$\xrightarrow{\text{change}} 5^{\frac{1}{2} \times \log_2^2} = \sqrt{5}$