باران کاظمی　　یازدهم ۲　　تکلیف شماره ۲۴

(۱)

$$f(x)=3^{Ax+B} \qquad y=x^2 \begin{cases} \xrightarrow{x=1} y=1 \to 3^{A+B}=1 \\ \xrightarrow{x=3} y=9 \to 3^{3A+B}=9 \end{cases} \qquad \begin{cases} A+\cancel{B}=0 \\ 3A+B=2 \to B=-1 \end{cases}$$

$$2A=2 \to A=1$$

$$f(x)=3^{x-1} \xrightarrow{x=0} 3^{-1}=\boxed{\frac{1}{3}}$$

(۲)

$$\log_2^{(4^x+15)}=x+3 \to 2^{x+3}=4^x+15 \to 2^x\times 8=2^{2x}+15 \xrightarrow{2^x=t}$$

$$8t=t^2+15 \to t^2-8t+15=0 \xRightarrow{(t-3)(t-5)=0} \begin{cases} t=3 \to 2^x=3 \to \log_2^3=x \\ t=5 \to 2^x=5 \to \log_2^5=x \end{cases}$$

$$\log_2^3+\log_2^5=\boxed{\log_2^{15}}$$

(۳)

$$(\log_{21}^3)^2+\log_{21}^{(147)}\log_{21}^{(1323)}=(\log_{21}^3)^2+(1+\log_{21}^7)(2+\log_{21}^3)$$

$$1323=21^2\times 3 \qquad 147=21\times 7$$

$$\log_{21}^3+\log_{21}^7=\underbrace{\log_{21}^{21}}_{1} \to \log_{21}^3+\log_{21}^7=1 \to \log_{21}^7=1-\log_{21}^3$$

$$\to (\log_{21}^3)^2+(\underbrace{2-\log_{21}^3)(2+\log_{21}^3)}_{4-(\log_{21}^3)^2}=\cancel{(\log_{21}^3)^2}+4-\cancel{(\log_{21}^3)^2}=\boxed{4}$$

(۴)

$$\log(x^2-2x+1)+3\log(1-x)=5 \qquad \log_3^{(-x)}=?$$

$$\log(1-x)^2+3\log(1-x)=5 \to \log(1-x)^2+\log(1-x)^3=5$$

$$\log(1-x)^2(1-x)^3=5 \to \log^{(1-x)^5}=5 \to 1-x=10 \to x=-9$$

$$\log_3^9=\boxed{2}$$

(۵)

$$\log_2^{(x^2+2x+4)}+\log_2^{(x-2)}=3 \qquad \log_{\sqrt[3]{2}}^x=?$$

$$\log_2^{(x^2+2x+4)(x-2)}=3 \Rightarrow \log_2^{x^3-8}=3 \to x^3-8=8 \to x^3=16 \Rightarrow x=\sqrt[3]{16}$$

$$\log_{\sqrt[3]{2}}^{\sqrt[3]{16}}=\log_{2^{\frac{1}{3}}}^{2^{\frac{4}{3}}}=\boxed{4}$$

(۶)

$$\log(2-x)-\log\frac{1}{(x-2)^2}=3 \qquad \log_{\sqrt{2}}^{(-x)}=?$$

$$\to \log\frac{(2-x)}{\frac{1}{(x-2)^2}}=\log(2-x)^3=3 \to 10=2-x \to x=-8$$

$$\log_{\sqrt{2}}^8=\frac{3}{\frac{1}{2}}=\boxed{6}$$

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$3^{x^2-2} = \cancel{81^x}\ 3^{4x}$ $\qquad \log_{6}(x-2) = ?$

$x^2-2 = 4x \rightarrow x^2-4x-2=0 \rightarrow x = \dfrac{4 \pm \sqrt{\cancel{16+8}\ 24}^{\ 2\sqrt6}}{2} = 2 \pm \sqrt6 \begin{cases} 2+\sqrt6 \\ 2-\sqrt6 \end{cases}$

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$\log_6 (2+\sqrt6-2) = \log_6 6^{\frac{1}{2}} = \boxed{\dfrac{1}{2}}$

(۸

$\log_3 2 = \dfrac{5}{8}$ $\qquad \log_{18} 8 = ?$

$\log_{18} 8 = 3\log_{18} 2 = \dfrac{3}{\log_2 18} = \dfrac{3}{\underbrace{\log_2 2}_{1} + 2\log_2 3} = \dfrac{3}{1+\frac{16}{5}} = \dfrac{3}{\frac{21}{5}} = \dfrac{15}{21} = \boxed{\dfrac{5}{7}}$

(۹

$\log_4 3 = 0.8$

$\frac{1}{2}\log_2 3 = 0.8 \rightarrow \log_2 3 = 1.6$ $\qquad \log_{12} 6 = ?$

$\log_{12} 6 = \log_{12} 2 + \log_{12} 3 = \dfrac{1}{\log_2 12} + \dfrac{1}{\log_3 12} = \dfrac{1}{\log_2 3 + 2} + \dfrac{1}{2\log_3 2 + 1}$

$= \dfrac{1}{3.6} + \dfrac{1}{\frac{18}{8}} = \dfrac{10}{36} + \dfrac{8}{18} = \dfrac{10+16}{36} = \dfrac{26}{36} = \boxed{\dfrac{13}{18}}$

(۱۰

$(a\log 2)x^2 + ax + b\log 2 = 0 \xrightarrow[-1]{\text{ریشه}} a\log 2 + b\log 2 = a \rightarrow b\log 2 = a - a\log 2$

$b\log 2 = a(1-\log 2) \rightarrow \dfrac{b}{a} = \dfrac{1-\log 2}{\log 2} \rightarrow \dfrac{\log 10 - \log 2}{\log 2} = \dfrac{\log \frac{10}{2}}{\log 2} = \dfrac{\log 5}{\log 2} = \log_2 5$

$(\sqrt2)^{\frac{b}{a}} = (\sqrt2)^{\log_2 5} = 5^{\frac{1}{2}\log_2 2} = \boxed{\sqrt5}$

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