باران کاظمی — یازدهم C — تکلیف شماره ۱۴

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۱)

$$f(x) = 3^{Ax+B} \qquad y = x^2 \begin{cases} \xrightarrow{x=1} y=1 \to 3^{A+B} = 1 \\ \xrightarrow{x=3} y=9 \to 3^{3A+B} = 9 \end{cases} \qquad \begin{cases} A+B = 0 \\ 3A+B = 2 \to B = -1 \end{cases} \quad 2A = 2 \to A = 1$$

$$f(x) = 3^{x-1} \xrightarrow{x=0} 3^{-1} = \boxed{\frac{1}{3}}$$

۲)

$$\log_2(4^x+15) = x+3 \to 2^{x+3} = 4^x+15 \to 2^x \times 8 = 2^{2x}+15 \xrightarrow{2^x=t}$$

$$8t = t^2+15 \to t^2-8t+15=0 \xrightarrow{(t-3)(t-5)=0} \begin{cases} t=3 \to 2^x=3 \to \log_2 3 = x \\ t=5 \to 2^x = 5 \to \log_2 5 = x \end{cases}$$

$$\log_2 3 + \log_2 5 = \boxed{\log_2 15}$$

۳)

$$(\log_{21} 3)^2 + \log_{21}(147)\,\log_{21}(1323) = (\log_{21} 3)^2 + (1+\log_{21} 7)(2+\log_{21} 3)$$

$$1323 = 21^2 \times 3 \qquad 147 = 21 \times 7$$

$$\log_{21} 3 + \log_{21} 7 = \log_{21} 21 = 1 \to \log_{21} 3 + \log_{21} 7 = 1 \to \log_{21} 7 = 1 - \log_{21} 3$$

$$\to (\log_{21} 3)^2 + \underbrace{(2-\log_{21} 3)(2+\log_{21} 3)}_{4-(\log_{21} 3)^2} = \cancel{(\log_{21} 3)^2} + 4 - \cancel{(\log_{21} 3)^2} = \boxed{4}$$

۴)

$$\log(x^2-2x+1) + 3\log(1-x) = 5 \qquad \log_3(-x) = ?$$

$$\log(1-x)^2 + 3\log(1-x) = 5 \to \log(1-x)^2 + \log(1-x)^3 = 5$$

$$\log(1-x)^2(1-x)^3 = 5 \to \log(1-x)^5 = 5 \to 1-x = 10 \to x = -9$$

$$\log_3 9 = \boxed{2}$$

۵)

$$\log_2(x^2+2x+4) + \log_2(x-2) = 3 \qquad \log_{\sqrt[3]{2}} x = ?$$

$$\log_2 (x^2+2x+4)(x-2) = 3 \Rightarrow \log_2 (x^3-8) = 3 \to x^3-8 = 8 \to x^3 = 16 \Rightarrow x = \sqrt[3]{16}$$

$$\log_{\sqrt[3]{2}} \sqrt[3]{16} = \log_{2^{\frac{1}{3}}} 2^{\frac{4}{3}} = \boxed{4}$$

۶)

$$\log(2-x) - \log\frac{1}{(x-2)^2} = 3 \qquad \log_{\sqrt{2}}(-x) = ?$$

$$\to \log\frac{(2-x)}{(x-2)^{-2}} = \log(2-x)^3 = 3 \to 10 = 2-x \to x = -8$$

$$\log_{\sqrt{2}} 8 = \frac{3}{\frac{1}{2}} = \boxed{6}$$

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$$3^{x^2-2} = \cancel{81^x}\ 3^{4x} \qquad \log_6^{(x-2)} = ?$$

$$x^2-2 = 4x \rightarrow x^2-4x-2=0 \rightarrow x = \frac{4 \pm \sqrt{\cancel{16+8}\ \overset{2\sqrt{6}}{\cancel{24}}}}{2} = 2 \pm \sqrt{6} \begin{cases} 2+\sqrt{6} \\ 2-\sqrt{6} \end{cases}$$

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$$\log_6^{2+\sqrt{6}-2} = \log_6^{6^{\frac{1}{2}}} = \boxed{\frac{1}{2}}$$

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$$\log_3^2 = \frac{5}{8} \qquad \log_{18}^8 = ?$$

$$\log_{18}^8 = 3\log_{18}^2 = \frac{3}{\log_2^{18}} = \frac{3}{\log_2^2 + 2\log_2^3} = \frac{3}{1+\frac{16}{5}} = \frac{3}{\frac{21}{5}} = \frac{15}{21} = \boxed{\frac{5}{7}}$$

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$$\log_4^3 = 0{,}8 \qquad \frac{1}{2}\log_2^3 = 0{,}8 \rightarrow \log_2^3 = 1{,}6 \qquad \log_{12}^6 = ?$$

$$\log_{12}^6 = \log_{12}^2 + \log_{12}^3 = \frac{1}{\log_2^{12}} + \frac{1}{\log_3^{12}} = \frac{1}{\log_2^3+2} + \frac{1}{2\log_3^2+1}$$

$$= \frac{1}{3{,}6} + \frac{1}{\frac{18}{8}} = \frac{10}{36} + \frac{8}{18} = \frac{10+16}{36} = \frac{26}{36} = \boxed{\frac{13}{18}}$$

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$$(a\log 2)x^2 + ax + b\log 2 = 0 \xrightarrow{\text{ریشه } -1} a\log 2 + b\log 2 = a \rightarrow b\log 2 = a - a\log 2$$

$$b\log 2 = a(1-\log 2) \rightarrow \frac{b}{a} = \frac{1-\log 2}{\log 2} \rightarrow \frac{\log 10 - \log 2}{\log 2} = \frac{\log\frac{10}{2}}{\log 2} = \frac{\log 5}{\log 2} = \log_2^5$$

$$(\sqrt{2})^{\frac{b}{a}} = (\sqrt{2})^{\log_2^5} = 5^{\frac{1}{2}\log_2^2} = \boxed{\sqrt{5}}$$