(۲۰)

دریا دیانی درودشتی

① ص

$f(x) = 3^{Ax+B}$

$y = x^2 \longrightarrow x=1 \to 1 = 3^{A+B}$

$\quad \longrightarrow 9 = 3^{3A+B}$

$\left.\begin{array}{l} A+B=0 \\ 3A+B=2 \end{array}\right\} 2A=2 \to A=1, \quad B=-1$

$3^{-1} \Rightarrow \frac{1}{3}$ [illegible]

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② ص

$\log_2 (4^x+15) = x+3 \longrightarrow 4^x+15 = 2^{x+3} \longrightarrow 2^{2x}+15 = 2^x \times 8 \qquad 2^x = t$

$t^2 - 8t + 15 \Rightarrow (t-5)(t-3) = 0 \to t=5 = 2^x \to x = \log_2 3$

$\qquad \qquad \qquad \qquad \hookrightarrow t = 3 = 2^x \to x = \log_2 5$

$\log_2 3 + \log_2 5 = \boxed{\log_2 15}$

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③ ص

$(\log_{21} 3)^2 + \log_{21} 147 \log_{21} 1323$

$(\log_{21} 3)^2 + (\log_{21} 7 + \log_{21} 21)(\log_{21} 3 + 2\log_{21} 21) \Rightarrow \log_{21} 7 = 1 - \log_{21} 3$

$(\log_{21} 3)^2 + (2 - \log_{21} 3)(2 + \log_{21} 3) \to (\log_{21} 3)^2 + 4 - (\log_{21} 3)^2 = ④$

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④ ص

$\log (x^2-2x+1) + 3\log(1-x) = 5$

$5\log(1-x) = 5 \longrightarrow 1-x = 10 \to x = -9 \qquad \log_3 9 = 2$

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⑤ ص

$\log_2 (x^2+2x+4) + \log_2 (x-2) = 3$

[illegible] $\Rightarrow \log_2 (x^2+2x+4) + \log_2 (x-2) \Rightarrow \log_2 (x^3-8) = 3$

$x^3 - 8 = 8 \to x^3 = 16 \Rightarrow x = \sqrt[3]{16} \longrightarrow \log_{\sqrt[3]{2}} \sqrt[3]{16} = \dfrac{\frac{4}{3}}{\frac{1}{3}} = ④$

(6)

$$\log(2-x) - \log\frac{1}{(x-2)^2} = 3$$

$$\log^{2-x} - \log(2-x)^{-2} = 3 \Rightarrow 3\log^{2-x} = 3 \rightarrow \log_{10}^{2-x} = 1$$

$$2 - x = 10 \rightarrow x = -8$$

$$\log_{\sqrt2}^{(-x)} = \log_{2^{\frac12}}^{2^3} \Rightarrow 6\log_2^2 = (6)$$

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(7)

$$3^{x^2-2} = 3^{4x} \rightarrow x^2 - 2 - 4x = 0 \rightarrow (x-2)^2 = 6$$

$$x - 2 = \sqrt6 \rightarrow x = \boxed{2+\sqrt6}$$

$$x - 2 = -\sqrt6 \rightarrow x = 2 - \sqrt6 \rightarrow \text{غ ق ق}$$

$$\log_6^{2+\sqrt6-2} = \log_6^{\sqrt6} = \boxed{\frac12}$$

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(8)

$$\log_{?}^{?} = \frac{\log_2^8}{\log_2^{18}} = \frac{3}{\log_2^2 + \log_2^9} = \frac{3}{1+2\log_2^3} = \frac{3}{1+2(\frac85)} = \frac{3}{\frac{21}{5}} = \boxed{\frac57}$$

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(9)

$$\log_4^3 = \frac{8}{10} \rightarrow \log_2^3 = 1.6$$

$$\log_{12}^6 = \frac{\log_2^6}{\log_2^{12}} = \frac{\log_2^3 + 1}{\log_2^3 + 2} = \frac{1.6+1}{1.6+2} = \frac{2.6}{3.6} = \frac{1.3}{1.8} = \boxed{\frac{13}{18}}$$

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(10)

$$(a\log 2)x^2 + ax + b\log 2 = 0$$

$$x = -1 \rightarrow a\log 2 - a + b\log 2 = 0 \rightarrow \log 2 = y \rightarrow ay - a + by = 0$$

$$y - 1 + \frac{b}{a}y = 0 \rightarrow y - \frac{b}{a}y = 1 \rightarrow 1 - \frac{b}{a} = \frac{1}{\log 2}$$

$$1 - \frac{b}{a} = \log_2^{10} \rightarrow 1 - \frac{b}{a} = 1 + \log_2^5 \Rightarrow \frac{b}{a} = -\log_2^5$$

$$\sqrt2^{-\log_2^5} = 5^{-\frac12} = \boxed{\sqrt5}$$