حلیا صفرپوران

-۱ $f(x)=3^{Ax+B} \to f(1)=3^{x-1} \xrightarrow{x=0} f' = \frac{1}{3} = y_0$; $y=x^2$

حل ۰.۱ → $y=x^2 \xrightarrow{x=1} 1^2=1 \to y=1$ (1,1)

نقطه تابع f با بعد y ها؟ $\log y = x^2 \xrightarrow{x=3} 9=y$ (3,9)

نقطه (1,1) → $f(x)=3^{A+B} \to A+B=0 \to A=-B$ / نقطه (3,9) → $9=3^{3A+B} \to 3A+B=2$

$-2B=2$, $B=-1$, $A=1$

$\log_2(4^x+15)=x+3$ $2^x=5 \to \log_2^5=x$; $2^x=3 \to \log_2^3=x$

$4^x+15=2^{(x+3)} \to 2^{2x}+3\times5=2^{(x+3)}$

$2^{2x}-8\times2^x+15=0 \to t^2-8t+15=0 \to (t-5)(t-3)=0 \to t=5$ یا $t=3$

$(2^x)^2$, t

-۳ حل: $(\log_{21}3)^2+\log_{21}(147)\cdot\log_{21}1323=(\log_{21}3)^2+4-(\log_{21}3)^2=4$

$(\log_{21}21\times7)(\log_{21}21^2\times3)$

$(1+\log_{21}7)(2+\log_{21}3)=(2-\log_{21}3)(2+\log_{21}3)=4-(\log_{21}3)^2$

$(1+1-\log_{21}3)$

-۴ $\log(x^2-2x+1)+3\log(1-x)=5$ $\log_3^{(-x)}=\log_9^{(x+4)}=2$

$\log(x-1)^2+3\log(1-x)=5 \to \log(1-x)^2+3\log(1-x)=5 \to 5\log(1-x)=5$

$2\log(1-x)$ $\log_{10}(1-x)=1$

$1-x=10 \to x=-9$

$\log_2(x^2+2x+4)+\log_2(x-2)=3$ $\log_{\sqrt[3]{2}}x-\log_{2^{1/3}}x=\frac{1}{3}\log_2 x=3\times2=6$

$\log_2(x^2+2x+4)\times(x-2)=3 \to x^3+2x^2+4x-2x^2-4x-8=8$

$x^3=16 \to x=4$

حل المسائل

۶- $\log(2-x) - \log\frac{1}{(x-2)^3} = 3 \qquad \log_{\sqrt{3}}^{-x}$

$\log\frac{1}{(x-2)^3} = \log(x-2)^{-3} = -\log(x-2)^3 = -\log(2-x)^3 = -3\log(2-x)$

$\log(2-x) - (-3\log(2-x)) = 3\log(2-x) = 3 \to \log(2-x) = 1 \to 2-x = 10$

$\boxed{x = -8}$

۷- $3^{x^2-2} = 81^x \qquad \log_4^{(x-3)} \geq 0 \to x > 3$

$= \log_4^{\sqrt{4}} = \boxed{\frac{1}{2}}$

$3^{x^2-2} = 3^{4x} \to x^2 - 2 = 4x \to x^2 - 4x - 2 = 0 \xrightarrow{+4-4=0} x^2 - 4x \underbrace{-2+4}_{+2} - 4 = 0$

$(x-2)^2 = 6 \xrightarrow{\text{رادیکال}} |x-2| = \sqrt{6} \to x-2 = \sqrt{6} \qquad (x-2)^2 - 6 = 0$

طبق فرض $x>3$

۸- $\log_5^2 = \frac{5}{8} \to \log_2^5 = (\log_5^2)^{-1} = \frac{8}{5} \qquad \log_{18}^8 = ? \qquad \frac{5}{7}$

$\log_{18}^8 = \frac{\log_2^8}{\log_2^{18}} = \frac{3}{\log_2^{3^2\times 2}} = \frac{3}{1+2\log_2^3} = \frac{3}{4.2} = \frac{5}{7}$

$\frac{16}{5} = 3.2$

۹- $\log_4^5 = 0.8 \qquad \log_{12}^6 = ? \quad 1.425$

$\log_4^{3} + \log_5^{2} = \log_{12}^{6} \to \log_2^{3} = ? \to \frac{1}{2}\log_2^{3} = 0.8 \to \log_5^{2} = \frac{5}{8}$

0.8

0.625

1.425

۱۰- $(a\log 2)x^2 + ax + b\log 2 = 0 \qquad (\sqrt{2})^{\frac{b}{a}}$

ضرب ریشه‌ها: -1

$P = -\frac{c}{a} = -\frac{b\log 2}{a\log 2} = \frac{-b}{a}$

حلیا جعفریان

$$\beta = -\frac{b}{a} \xrightarrow[x = -\frac{b}{a}]{\text{در معادله صدق می‌کند}} \left(-\frac{b}{a}\right)^2 a\log 2 + -\frac{b}{a}\times a + b\log 5 = 0$$ ۱۵

$$\frac{b^2}{a}\log 2 - b + b\log 5 = 0 \to \log 2\left(\frac{b^2}{a}+b\right) = b \xrightarrow{\div b} \log 2\left(\frac{b}{a}+1\right) = 1$$

$$\log 2 = \frac{1}{\frac{b}{a}+1} \xrightarrow{\text{معکوس}} \log_2 10 = \frac{b}{a}+1 \to \log_2 10 - \underbrace{1}_{\log_2 2} = \frac{b}{a} \to \log_2 5 = \frac{b}{a}$$

$$\sqrt{2}^{\frac{b}{a}} = \sqrt{2}^{\log_2 5} \;(\text{تعویض}) = 5^{\log_2 \sqrt{2}} = 5^{\frac{1}{2}} = \sqrt{5}$$