حسابان ریاضی / تکلیف ۲۴ / یازدهم C

/ /

1

$f(x) = 3^{Ax+B}$ و $y = x^2$ } تقاطع $\xrightarrow{(1,1)}$ $(3,9)$

$\to 3^{A+B} = 1 \to A+B = 0 \to A = -B$

$3^{3A+B} = 9 \xrightarrow{A=-B} 3^{2A} = 3^2 \to A = 1 , B = -1$

عرض از مبدأ $f(x)$ ، $x = 0$ } ؟ $\quad 3^{x-1} \xrightarrow{x=0} (0,y) \quad 3^{-1} = \frac{1}{3}$

2

$\log_2 (4^x + 15) = x + 3 \to \underbrace{2^{x+3}}_{8\times 2^x} = 2^{2x} + 15 \xrightarrow{2^x = t} t^2 - 8t + 15 = 0 \to (t-5)(t-3)$

$t = 5 , 3$

مجموع جواب‌های معادله = ؟

$2^x = 5$ و $2^x = 3$

$x = \log_2 5$, $x = \log_2 3$ $\longrightarrow$ $\log_2 5 + \log_2 3 = \log_2 15$

3

$(\log_{21} 3)^2 + \log_{21}(147) \times \log_{21}(1323)$

$\log_{21}(147) = 1 + \log_{21} 7 = 1 - \log_{21} 3 \to (2 - \log_{21} 3)$

$\log_{21}(1323) = \log_{21} 21^2 \times 3 = (2 + \log_{21} 3)$

مزدوج‌ها $4 - (\log_{21} 3)^2$

$\to (\log_{21} 3)^2 + 4 - (\log_{21} 3)^2 \to 4$

4

$\log (x^2 - 2x + 1) + 3\log (1-x) = 5$

$x^2 - 2x + 1 = (x-1)^2$, $3\log(1-x) = \log(1-x)^3$

$\to (x-1)^2 \times -(x-1)^3 = 10^5$

$-(x-1)^5 = 10^5 \to -x + 1 = 10$

$-9 = x$

$\log_3 (-x) = ؟$

$\to \log_3 -(-9) = \log_3 9 = 2$

5

$\log_2 (x^2 + 2x + 4) + \log_2 (x - 2) = 3$

$\to (x^2 + 2x + 4)(x - 2) = 8$

$x^3 + 2x^2 + 4x - 2x^2 - 4x - 8 = 8$

$\log_{\sqrt[3]{2}} x = ؟$

$x^3 = 16 \to x = 2^{\frac{4}{3}}$

$\log_{2^{\frac{1}{3}}} 2^{\frac{4}{3}} = 4$

6

$\log^{(2-x)} - \log^{\frac{1}{(2-x)^2}} = 4$

$\hookrightarrow \log^{\frac{2-x}{\frac{1}{(x-2)^2}}} = 4 \rightsquigarrow \frac{-(x-2)(x+2)^2}{-(x-2)^2} = 9^4 \hookrightarrow \frac{x^2-4}{x-2} = 9^4$

$\log^{(-x)}_{\sqrt{2}} = ?$

$\log_{10}$

$\frac{1}{2}\log_2^4 = \frac{1}{2}\times 2 = 1$

$\log_{\sqrt{2}} = 4$

7

$4^{x^2-2x} = 4^{x+4} \rightsquigarrow (x^2-4x-12=0)_{+4}$

$\frac{x^2-4x-4}{(x-2)^2} = 4 \rightarrow x = 2\sqrt{4}$

$\log_4^{(x-2)} = ? \hookrightarrow \log_4^{\sqrt{4}} = \frac{1}{2}$

8

$\log^{8}_{18} = ? \hookrightarrow \frac{\log^8_2}{\log^{18}_2} = \frac{3}{\log^2_3 + 1} \rightsquigarrow \frac{3}{2,4} = \frac{30}{24} = \frac{5}{4}$

$\log^2_3 = \frac{5}{8}$

$\log^3_2 = 1,4$

9

$\log^3_4 = 0,8 \rightsquigarrow \log^4_3 = 1,25 \rightsquigarrow 2\log^2_3 = 1,25 \rightsquigarrow \log^2_3 = 0,625$

$\log^4_{12} = ? \hookrightarrow \frac{\log^4_3}{\log^{12}_3} = \frac{\log^2_3}{1+2\log^2_3} = \frac{0,625}{2,25} = \frac{625}{2250} = \frac{25}{90} = \frac{5}{18}$

$\frac{13}{18}$

10

$(\underbrace{a\log^2_{10}}_{a'})x^2 + \underbrace{a}_{b'}x + \underbrace{b\log^2_{10}}_{c'} = 0 \rightarrow -\frac{c'}{a'}$

$a'+c'=2b'$

$\hookrightarrow a\log^2_{10} + b\log^2_{10} = a$

$b\log^2_{10} = a - a\log^2_{10}$

$a(1-\log^2_{10})$

$\log^5_{10}$

$(\sqrt{2})^{\frac{b}{a}} = ?$

$\hookrightarrow (\sqrt{2})^{\log^5_2}$

$\hookrightarrow 2^{\frac{1}{2}\log^5_2} \hookrightarrow 2^{\log^{\sqrt{5}}_2} \rightarrow \sqrt{5}$

$\frac{b}{a} = \frac{\log^5_{10}}{\log^2_{10}} = \log^5_2$

Parsian