1 (۲)

$f(x) = 3^{Ax+B}$, $y = x^2$ — تقاطع $(1,1)$, $(3,9)$

$\to 3^{A+B} = 1 \to A+B = 0 \to A = -B$

$\to 3^{3A+B} = 9 \xrightarrow{A=-B} 3^{2A} = 3^2 \to A = 1 , B = -1$

عرض از مبدا $f(x), x=0$? $\quad 3^{x-1} \xrightarrow{(x=0)} (0,?) \quad 3^{-1} = \frac{1}{3}$

2 (۳)

$\log_2(4^x + 15) = x+3 \to \underbrace{2^{x+3}}_{8\times 2^x} = 2^{2x} + 15 \xrightarrow{2^x = t} t^2 - 8t + 15 = 0 \to (t-5)(t-3)$

$t = 5 , 3$

$2^x = 5 , 2^x = 3$

مجموع جواب های معادله = ?

$\log_2 5 + \log_2 3 = \log_2 15 \leftarrow x = \log_2 5 , x = \log_2 3$

3 (۳)

$(\log_{21} 3)^2 + \log_{21}(147) \times \log_{21}(1323)$

$\log_{21}(147) = 1 + \log_{21} 7 = 1 + 1 - \log_{21} 3 = (2 - \log_{21} 3)$

$\log_{21}(1323) = \log_{21} 21^2 \times 3 = (2 + \log_{21} 3)$

ها مزدوج $\quad 4 - (\log_{21} 3)^2$

$\hookrightarrow (\log_{21} 3)^2 + 4 - (\log_{21} 3)^2 \to 4$

4 (۲)

$\log \frac{(x^2 - 2x + 1)}{(x-1)^2} + 3\log(1-x) = 5$

$3\log(1-x) = \log(1-x)^3 \hookrightarrow (x-1)^2 \times -(x-1)^3 = 10^5$

$-(x-1)^5 = 10^5 \to -x+1 = 10$

$-9 = x$

$\log_3(1-x) = ?$

$\hookrightarrow \log_3 -(-9) = 2$

5 (۳)

$\log_2(x^2 + 2x + 4) + \log_2(x-2) = 3$

$\hookrightarrow (x^2 + 2x + 4)(x-2) = 8$

$x^3 + 2x^2 + 4x - 2x^2 - 4x - 8 = 8$

$\log_{\sqrt[3]{2}} x = ?$

$x^3 = 16 \to x = 2^{\frac{4}{3}}$

$\log_{2^{\frac{1}{3}}} 2^{\frac{4}{3}} = 4$

6

$\log(2-x) - \log \frac{1}{(x-2)^2} = 3$

$\hookrightarrow \log_{10} \frac{2-x}{\frac{1}{(x-2)^2}} = 3 \rightsquigarrow -(x-2)(x-2)^2 = 10^3$

$-(x-2)^3 = 10^3 \hookrightarrow x-2 = -10$, $x = -8$

$\log_{\sqrt{2}}(-x) = ?$

$\frac{3}{\frac{1}{2}} \log_2 2 = 6$

$\log_{\sqrt{2}} -(-8) = 6$

7

$3^{x^2-2} = 3^{4x} \rightsquigarrow (x^2 - 4x - 2 = 0) \xrightarrow{+6 \quad +6} \underbrace{x^2 - 4x + 4}_{(x-2)^2} = 6 \rightarrow x-2 = \sqrt{6}$

$\log_6 (x-2) = ? \hookrightarrow \log_6 \sqrt{6} = \frac{1}{2}$

8

$\log_{18} 8 = ? \hookrightarrow \frac{\log_2 8}{\log_2 18} = \frac{3}{1+2\log_2 3} \rightsquigarrow \frac{3}{4.2} = \frac{30}{42} = \frac{5}{7}$

$\log_3 2 = \frac{5}{8}$

$\log_2 3 = 1.6$

9

$\log_4 3 = 0.8 \rightsquigarrow \log_3 4 = 1.25 \rightsquigarrow 2\log_3 2 = 1.25 \rightarrow \log_3 2 = 0.625$

$\log_{12} 6 = ? \hookrightarrow \frac{\log_3 6}{\log_3 12} = \frac{1+\log_3 2}{1+2\log_3 2} = \frac{1.625}{2.25} = \frac{1625 : 5}{2250 : 5} = \frac{325 : 25}{450 : 25} = \frac{13}{18}$

10

$\underbrace{(a \log_{10} 2)}_{a'} x^2 + \underbrace{ax}_{b'} + \underbrace{b \log_{10} 2}_{c'} = 0 \begin{cases} -1 \\ -\frac{c'}{a'} \end{cases}$ $a' + c' = b'$

$\hookrightarrow a\log_{10} 2 + b\log_{10} 2 = a$

$b\log_{10} 2 = a - a\log_{10} 2 = a(1-\log_{10} 2) = \log_{10} 5$

$\frac{b}{a} = \frac{\log_{10} 5}{\log_{10} 2} = \log_2 5$

$(\sqrt{2})^{\frac{b}{a}} = ?$

$\hookrightarrow (\sqrt{2})^{\log_2 5}$

$\hookrightarrow 2^{\frac{1}{2}\log_2 5} \hookrightarrow 2^{\log_2 \sqrt{5}} \rightarrow \sqrt{5}$