سارینا اسپنانی یازدهم دهم - شماره تکلیف (۲۴)

(۱)

$f(x) = 3^{Ax+B}$

$y = x^2$

طول‌ها $\rightarrow$ ۳ و ۱

$(3, 9) \quad (1, 1)$

$3^{A+B} = 1$

$3^{3A+B} = 9$

$\begin{cases} A+B=0 \\ 3A+B=2 \end{cases} \rightarrow \begin{cases} -A-B=0 \\ 3A+B=2 \end{cases}$

$2A = 2$

$\boxed{A=1}$

$A+B=0$

$\boxed{B=-1}$

$f(0) = 3^B = 3^{-1} = \frac{1}{3}$

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(۲)

$\log_2(4^x+15) = x+3$

$2^{x+3} = 4^x + 15$

$2^x \times 8 = (2^x)^2 + 15$

$0 = t^2 - 8t + 15 = 0$

$(t-5)(t-3) = 0$

$t=5 \quad t=3$

$2^x = 5 \quad 2^x = 3$

$x_2 = \log_2 5 \quad x_1 = \log_2 3$

$x_1 + x_2 = \log_2 5 + \log_2 3 = \log_2 15$

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(۳)

$(\log_{21} 3)^2 + \log_{21} 147 \log_{21} 1323 =$

$\log_{21} 147 = \log_{21} 21 \times 7 = 1 + \log_{21} \frac{21}{3} = 1 + \log_{21} 21 - \log_{21} 3 = 2 - \log_{21} 3$

$\log_{21} 1323 = \log_{21} 21^2 \times 3 = 2 + \log_{21} 3$

ضرب $= 4 - (\log_{21} 3)^2$

$\rightarrow (\log_{21} 3)^2 + 4 - (\log_{21} 3)^2 = 4$

(۴)

$\log^{x^2-2x+1} + 2\log^{(1-x)} = 0$

$\log^{(1-x)^2} + 2\log^{(1-x)} = 0$

$\underbrace{2\log^{(1-x)}}_{t} + 2\log^{(1-x)} = 0$

$\therefore t = 0$

$t = 1 \rightarrow \log_{10}^{(1-x)} = 1$

$1 - x = 10$

$\boxed{-9 = x}$

$\log_2^{(-x)} = ?$

$\hookrightarrow \log_2^9 = (2)$

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(۵)

$\log_2^{x^2+2x+2} + \log_2^{x-2} = 2$

$\log_2^{x^2+2x+2} + \log_2^{x-2} = 2$

$\log_2^{(x^2+2x+2)(x-2)} = 2$

$\log_2^{x^3-8} = \log_2^{8}$

$\sqrt[3]{x^3} = \sqrt[3]{16}$

$x = 2\frac{2}{3}$

$\log_{\sqrt{2}}^{x} = ?$

$\log_{2^{\frac{1}{2}}}^{2^{\frac{3}{2}}} = \frac{\frac{3}{2}}{\frac{1}{2}} \log_2^2 = (3)$

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(۶)

$\log^{(2-x)} - \log^{\frac{1}{(x-2)^2}} = 2$

$\log^{(2-x)} + \log^{(x-2)^2} = 2$

$\log^{-(x-2)} + \log^{(x-2)^2} = 2$

$\log_{10}^{-(x-2)(x-2)^2} = 2$

$-t(t^2) = 10^2$

$-t^3 = 10^2$

$\boxed{t = -10}$

$t = x - 2 = -10$

$\boxed{x = -8}$

$\log_{\sqrt{2}}^{(-x)} = \log_{2^{\frac{1}{2}}}^{8} = \log_{2^{\frac{1}{2}}}^{2^3} = (6)$

$3^{x^2-2} = 8^x$

$3^{x^2-2} = 2^{3x}$

$x^2 - 3x - 2 = 0$

$\Delta = b^2 - 4ac = 9 - 4(-2) = 17$

$\frac{3 \pm \sqrt{17}}{2} \longrightarrow \frac{3 \pm \sqrt{17}}{2} \longrightarrow \frac{3}{2} \pm \sqrt{\frac{17}{4}} = x$

$\alpha = \frac{3}{2} + \sqrt{\frac{17}{4}}$

$\beta = \frac{3}{2} - \sqrt{\frac{17}{4}}$

$\log_y x^{-2} = \log_y^{\frac{3}{2}+\sqrt{\frac{17}{4}}-\frac{3}{2}+\sqrt{\frac{17}{4}}} \; y = \frac{1}{2}$ (۷)

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(۸)

$\log_3^2 = \frac{a}{8}$

$\log_{12}^8 = \frac{\log_3^8}{\log_3^{12}} = \frac{3\log_3^2}{\log_3^{3^2 \times 2}} = \frac{3\log_3^2}{2 + \log_3^2} = \frac{3 \times \frac{a}{8}}{2 + \frac{a}{8}} = \frac{\frac{3a}{8}}{\frac{16+a}{8}} = \frac{3a}{16+a}$

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(۹)

$\log_4^3 = 0/8$

$\log_{12}^6 = \frac{\log_4^6}{\log_4^{12}} = \frac{\log_4^2 + \log_4^3}{\log_4^4 + \log_4^3} = \frac{\frac{1}{2} + \frac{8}{10}}{1 + \frac{8}{10}} = \frac{\frac{12}{10}}{\frac{18}{10}} = \frac{12}{18}$

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(۱۰)

$(a \log 2)x^2 + ax + b\log 2 = 0$

$\alpha + \beta = \frac{-b}{a}$

$1 - \frac{b}{a} = \frac{-c}{a} = \frac{-b\log 2}{a \log 2} = \frac{-b}{a}$

$1 - \frac{-b}{a} = -\log_2^{10}$

$\frac{b}{a} = \log_2^5$

$\sqrt{2}^{\log_2^5} = 5^{\log_2^{\sqrt{2}}} = 5^{\frac{1}{2}} = \sqrt{5}$