پارسی عباسیان یازدهم ریاضی تکلیف ۲۳

۱.

$\log_n m = a \qquad \log_m n = \frac{1}{a} \to \log_m n + \log_m m = \log_m mn = \frac{1}{a} + 1$

$\log_m mn = \frac{a+1}{a} \qquad \log_{mn} m = \frac{a}{a+1} \to \log_{mn} m^2 n = \log_{mn} m + \log_{mn} mn = b$

$\frac{a}{a+1} + 1 = b \to \left[\frac{a}{a+1}\right] + 1 = 1 = [b]$

۲.

الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}} x}} \to \log_{\frac{1}{2}} x > 0 \to x > 0 \quad \log_{\frac{1}{2}} x > 0 \to x < 1$

$D_y = (0, 1)$

ب) $y = \frac{\log_4 (x^2 - x - 2)}{\sqrt{x^2-1} + 1}$ $\to x^2 - x - 2 > 0$ (جدول علامت: -1، 2؛ $+\ -\ +$) $\to \sqrt{x^2-1} + 1 \neq 0 \qquad D_y = (-\infty, -1) \cup (2, +\infty)$

۳.

$2\log_x a + \log_a \sqrt{x} = 2 \to 2\log_x a + \frac{1}{2\log_x a} - 2 = 0$

$4(\log_x a)^2 + 1 - 4\log_x a = 0 \to (2\log_x a - 1)^2 \to 2\log_x a = 1$ ($x = 9$، $a = 3$)

۴.

$\to (\log 10 - \log 2 - \log 3)x^2 + (\log 3 + \log 3)x - (\log 10 - \log 2 + \log 3)$

$\log 2 \simeq 0.3$

$\log 3 \simeq 0.4$

$(1 - 0.3 - 0.4)x^2 + (0.4 + 0.4)x - (1 - 0.3 + 0.4)$

$(\log \frac{5}{3})x^2 + (\log 9)x - \log 15 = 0 \Rightarrow 0.3x^2 + 0.8x - 1.1 = 0$

$S = -\frac{8}{3} \quad P = -\frac{11}{3}$

$|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{\frac{64}{9} + \frac{44}{3}} = \sqrt{\frac{196}{9}} = \frac{14}{3}$

۵.

$\log_2 7 = 2.8 \to \log_2 7 + \log_2 2 = 3.8 = \log_2 14$

$\log_5 2 = 0.5 \to \log_2 5 = 2 \to \log_2 5 + \log_2 2 \to \log_2 10 = 3$

$\log_{14} 10 = \frac{15}{19}$

$\Rightarrow \frac{\log_2 10}{\log_2 14} = \frac{3}{3.8} = \frac{30}{38} = \frac{15}{19}$

۶.

$\log_{3}5 = 1.5 \rightarrow \log_{3}5 + \log_{3}3 = \log_{3}15 = 2.5$

$\log_{2}3 = 1.6 \rightarrow \log_{3}2 = \frac{5}{8} \rightarrow \log_{3}2 + \log_{3}3 = \log_{3}6 = \frac{13}{8}$

$\log_{15}6 = ? \longrightarrow \frac{\log_{3}6}{\log_{3}15} = \frac{\frac{13}{8}}{\frac{5}{2}} = \frac{13}{20}$

۷.

$\log_{8}18 = m \longrightarrow \frac{\log_{2}18}{\log_{2}8} = m \rightarrow \log_{2}18 = 3m$

$\log_{2}18 - \log_{2}2 = \log_{2}9 = 2\log_{2}3 \Rightarrow 3m-1 \rightarrow \log_{2}3 = \frac{3m-1}{2}$

$\log_{4}12 = \frac{3m+3}{4}$

$\log_{4}12 = \frac{\log_{2}12}{\log_{2}4} = \frac{\log_{2}3 + \log_{2}4}{2} = \frac{\frac{3m-1}{2}+2}{2} = \frac{3m+4-1}{4} = \frac{3m+3}{4}$

۸.

$(0.4)^{3x-1} = \left(\frac{125}{8}\right)^{x^2} \Rightarrow \left(\frac{2}{5}\right)^{3x-1} = \frac{2}{5}^{-3x^2} \rightarrow 3x^2+2x-1=0 \rightarrow x \begin{cases} -1 \\ \frac{1}{3} \end{cases}$

$\log_{8}^{9x+1} \rightarrow 9x+1>0 \rightarrow x>-\frac{1}{9} \rightarrow x=\frac{1}{3}$

$\log_{8}4 = \log_{2^3}2^2 = \frac{2}{3}$

۹.

$\log_{2}3 = a \rightarrow \log_{2}3 + \log_{2}2 = \log_{2}6 \rightarrow \frac{2}{3}(\log_{2}6) = \log_{8}36$

$\log_{8}b = \frac{2}{3}(1+a) \rightarrow b=36$

$\log 3b-8 \rightarrow \log_{10}100 = 2$

۱۰.

$a = \frac{b+c}{2} \rightarrow b = 2a-c \rightarrow \frac{1}{x_1+x_2} \div \frac{1}{S} = \frac{4a}{b} = \frac{4a}{2a-c} = 2\log 2$

$\frac{2a}{2a-c} = \log 2 \longrightarrow \frac{2a-c}{2a} = \log_{2}10 \rightarrow 1-\log_{2}10 = \frac{c}{2a} \rightarrow 2-\log_{2}100 = \frac{c}{a}$

$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} \Rightarrow \left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}} = \left(2^{\frac{c}{a}}\right)^{-\frac{1}{8}} \longrightarrow 2^{(2-\log_{2}100)^{-\frac{1}{8}}}$

$\left(\frac{2^2}{2^{\log_{2}100}}\right)^{-\frac{1}{8}} \Rightarrow \left(\frac{4}{100}\right)^{-\frac{1}{8}} = (25)^{\frac{1}{8}} \rightarrow \sqrt[4]{5}$