

$$\log_n m = a, \log_{mn} m^n = b, a > 0, [b] = ?$$

$$\log_{mn} m^n = b \Rightarrow \frac{\log m^n}{\log mn} = \frac{\log m^n + \log \frac{1}{n}}{\log m + \log \frac{1}{n}} = \frac{2 \log m + 1}{a+1} = \frac{2a+1}{a+1} = b \Rightarrow \left[\frac{2a+1}{a+1} \right] = [b]$$

$$\Rightarrow \left[\frac{a+a+1}{a+1} \right] = [b] \Rightarrow \left[\frac{a+1}{a+1} + \frac{a}{a+1} \right] \Rightarrow \left[1 + \frac{a}{a+1} \right] \Rightarrow 1 + \left[\frac{a}{a+1} \right] \xrightarrow{a > 0} 1 + 0 = 1$$

لجواب ۱

الف) $y = \sqrt{\frac{u}{\log u}} \rightarrow \frac{u}{\log u} > 0 \Rightarrow \frac{u}{\log u} > 0 \Rightarrow u \in (0, 1) \cup (1, \infty)$ و $\log \frac{1}{u} \neq 0 \Rightarrow u \neq 1$

ب) $y = \frac{\log(u^2 - u - 2)}{\sqrt{u^2 - 1} + 1}$ و $\log(u^2 - u - 2) \geq 0 \Rightarrow u^2 - u - 2 \geq 0 \Rightarrow (u-2)(u+1) \geq 0 \Rightarrow u \leq -1$ یا $u \geq 2$

جواب: $D_f = (0, 1) \cup (2, \infty)$

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جواب: $D_f = (-\infty, -1) \cup (2, \infty)$

$$2 \log_a u + \log_a \sqrt{u} = 2 \xrightarrow{u=9} 2 \log_a 9 + \log_a 3 = 2 \Rightarrow 2 \log_a 9 + \log_a 3 = 2 \Rightarrow \log_a 9 + \frac{1}{\log_a 9} = 2 \Rightarrow$$

$$\frac{(\log_a 9)^2 + 1}{\log_a 9} = 2 \Rightarrow (\log_a 9)^2 + 1 = 2 \log_a 9 \Rightarrow (\log_a 9)^2 - 2 \log_a 9 + 1 = 0 \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow$$

$$(t-1)^2 = 0 \Rightarrow t = 1 \Rightarrow \log_a 9 = 1 \Rightarrow a = 9$$

جواب: $a = 9$

$$(\log \frac{w}{r}) u^2 + (\log 9) u - \log 10 = 0 \Rightarrow (\log w - \log r) u^2 + (2 \log 3) u - (\log 10 + \log w) = 0$$

$$\textcircled{A} \log w - \log r = \log 10 - \log 2 - \log 3 = 1 - 0.18 - 0.18 = 0.64$$

$$\textcircled{B} 2 \times \frac{1}{10} = \frac{1}{5} = 0.2$$

$$\log 3 + \log 10 - \log 2 = 0.18 + 1 - 0.18 = 1$$

$$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{b^2 - 4ac}}{|a|} = \frac{\sqrt{1.94}}{1.018} = \frac{1.4}{1.018} = \left(\frac{14}{1018} \right) \rightarrow$$

$$\Delta = b^2 - 4ac = (0.18)^2 - (2 \times 0.18 \times -1) = 1.94$$

$$\frac{\log \frac{1}{r} \times u}{\log \frac{1}{r} \times u} = \frac{\log \frac{1}{r} + \log u}{\log \frac{1}{r} + \log u} = \frac{3}{4.18} = \frac{3}{4.18} = \left(\frac{15}{19} \right)$$

لجواب $\frac{15}{19}$

$$\log u = 2.18 \text{ و } \log w = 0.18 \Rightarrow \log \frac{w}{r} = 2$$

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$$\log_{10} r = \frac{\log r^{10}}{\log 10} = \frac{\log r + \log r + \log r + \log r + \log r}{1} = \frac{5 \log r}{1} = 5 \log r \Rightarrow \log r = \frac{1}{5} \log_{10} r$$

$$\log r = \frac{1}{5} \log_{10} r \Rightarrow \log r = \frac{1}{5} \log_{10} r$$

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$$\log_{10} r = m \Rightarrow \frac{1}{r} \log_{10} r^{r+1} \Rightarrow \frac{1}{r} (\log_{10} r + \log_{10} r) \Rightarrow \frac{1}{r} (2 \log_{10} r) = m \Rightarrow \frac{2}{r} \log_{10} r = m \Rightarrow \log_{10} r = \frac{r}{2} m$$

$$\frac{2}{r} \log_{10} r = m \Rightarrow \log_{10} r = \frac{r}{2} m$$

$$\log_{10} r^{r+1} = \log_{10} r + \log_{10} r = 1 + \log_{10} r = 1 + \frac{r}{2} m = \frac{r}{2} m + 1$$

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$$(10)^{ru-1} = \left(\frac{10}{r}\right)^{ru}, \log_{10} (ru+1) = ?$$

$$\left(\frac{10}{r}\right)^{ru-1} = \left(\frac{10}{r}\right)^{ru} \Rightarrow \left(\frac{10}{r}\right)^{ru-1} = \left(\frac{10}{r}\right)^{ru} \Rightarrow \left(\frac{10}{r}\right)^{1-ru} = \left(\frac{10}{r}\right)^{ru} \Rightarrow 1-ru = ru \Rightarrow ru = \frac{1}{2}$$

$$ru = \frac{1}{2} \Rightarrow u = \frac{1}{2r} \Rightarrow \log_{10} (ru+1) = \log_{10} \left(\frac{1}{2r} + 1\right) = \log_{10} \left(\frac{1+2r}{2r}\right) = \log_{10} (1+2r) - \log_{10} (2r)$$

$$\log_{10} (ru+1) = \log_{10} \left(\frac{1+2r}{2r}\right) = \log_{10} (1+2r) - \log_{10} (2r)$$

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$$\log_{10} b = \frac{r}{2} (1+a), \log_{10} r = a \Rightarrow \frac{1}{r} \log_{10} b = \frac{r}{2} (1+a) \Rightarrow \frac{1}{r} \log_{10} b = \frac{r}{2} + \frac{r}{2} \log_{10} r \Rightarrow \frac{1}{r} \log_{10} b - \frac{r}{2} \log_{10} r = \frac{r}{2} \Rightarrow \frac{1}{r} (\log_{10} b - r \log_{10} r) = \frac{r}{2} \Rightarrow \log_{10} \left(\frac{b}{r^r}\right) = \frac{r^2}{2} \Rightarrow \log_{10} \frac{b}{r^r} = \frac{r^2}{2} \Rightarrow \frac{b}{r^r} = 10^{\frac{r^2}{2}} \Rightarrow b = r^r \cdot 10^{\frac{r^2}{2}}$$

$$\log_{10} (rb-1) = \log_{10} (r \cdot 10^{\frac{r^2}{2}} - 1) = \log_{10} (10^{\frac{r^2}{2}}) = \frac{r^2}{2}$$

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$$\frac{a}{b} = \frac{r}{2} \log_{10} r \Rightarrow \frac{a}{b} = \log_{10} r \Rightarrow b = \frac{ra}{\log_{10} r} \Rightarrow \frac{b+c}{r} = a \Rightarrow c = ra - b$$

$$\frac{c}{a} = \frac{ra-b}{a} = r - \frac{b}{a} = r - \left(\frac{ra}{\log_{10} r} \times \frac{1}{a}\right) = r - \frac{r}{\log_{10} r} = r \left(1 - \frac{1}{\log_{10} r}\right) = r - r \log_{10} r$$

$$\frac{1}{r - r \log_{10} r} = \left(r - r \log_{10} r\right)^{-1} = \left(r - r \log_{10} r\right)^{-1} \Rightarrow r^{-\frac{1}{2} \log_{10} r} = r^{\log_{10} r} \Rightarrow \log_{10} r = \frac{1}{2} \log_{10} r \Rightarrow \log_{10} r = \frac{1}{2} \log_{10} r$$

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