۱-

$$\log_{nm}^{m^2 n} = \frac{\log_n^{m^2 n}}{\log_n^{nm}} = \frac{\log_n^{m^2} + \log_n^n}{\log_n^m + \log_n^n} =$$

$$\frac{2\log_n^m + 1}{\log_n^m + 1} = \frac{2a+1}{a+1} = b$$

$$[b] = \left[\frac{a+a+1}{a+1}\right] = \left[\frac{a}{a+1} + 1\right] = \left[\frac{a}{a+1}\right] + 1$$

$$\xrightarrow{a>0} [b] = 0 + 1 = \boxed{1}$$

۲- الف)

$$y = \sqrt{\frac{x}{\log_{\frac{1}{2}}^x}} \qquad x > 0$$

$\frac{x}{\log_{\frac{1}{2}}^x} \geq 0 \rightarrow \log_{\frac{1}{2}}^x$ و x باید هم علامت باشند

$$\Rightarrow \log_{\frac{1}{2}}^x \geq 0 \Rightarrow x \leq 1 \Rightarrow D_f = (0,1]$$

ب)

$$y = \frac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1} \qquad x^2 - 1 \geq 0 \Rightarrow$$

$$x^2 \geq 1 \Rightarrow x \geq 1$$

$$x \leq -1 \quad ①$$

$x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow$

$$\begin{array}{ccccc} & -1 & & 2 & \\ \hline + & \circ & - & \circ & + \end{array} \qquad \begin{array}{l} x < -1 \\ x > 2 \end{array} \quad (2)$$

$\xrightarrow{(1),(2)} D_f = (-\infty, -1) \cup (2, +\infty)$

۳-

$2\log_x a + \log_a \sqrt{x} = 2 \xrightarrow{x=9}$

$2\log_9 a + \log_a 3 = 2$

$\log_{9 \to 3^2} a = \frac{1}{2}\log_3 a = \frac{1}{2\log_a 3}$

$\Rightarrow 2 \times \frac{1}{2\log_a 3} + \log_a 3 = 2 \xrightarrow[\log_a 3 = t]{}$

$t + \frac{1}{t} = 2 \Rightarrow \frac{t^2+1}{t} = 2 \Rightarrow t^2 - 2t + 1 = 0$

$(t-1)^2 = 0 \Rightarrow t = 1 \Rightarrow \log_a 3 = 1 \Rightarrow \boxed{a = 3}$

۴-

$\log \frac{5}{3} = \log 5 - \log 3 = 0.7 - 0.4 = 0.3$

$\log 5 \downarrow \; \log \frac{10}{2} = \log 10 - \log 2 = 1 - 0.3 = 0.7$

$\log 9^{\to 3^2} = 2\log 3 = 2 \times 0,4 = 0,8$

$\log 15^{\to 5\times 3} = \log 5 + \log 3 = 0,7 + 0,4 = 1,1$

$\Rightarrow 0,3x^2 + 0,8x - 1,1 = 0 \xrightarrow{\times 10}$

$3x^2 + 8x - 11 = 0 \xrightarrow{\text{وسط}} x^2 + 8x - 33 = 0$

$(x+11)(x-3) = 0 \rightarrow x = \frac{3}{3} = 1$

$\rightarrow x = \frac{-11}{3}$

اختلاف $= 1 - \left(\frac{-11}{3}\right) = 1 + \frac{11}{3} = \boxed{\frac{14}{3}}$

$\log_5^2 = \frac{\log_2^2}{\log_2^5} = \frac{1}{\log_2^5} = \frac{1}{2} \Rightarrow \log_2^5 = 2$ -۵

$\log_{14}^{10} = \frac{\log_2^{10}}{\log_2^{14}} = \frac{\log_2^5 + \log_2^2}{\log_2^7 + \log_2^2} = \frac{2+1}{2,8+1} =$

$\frac{3}{3,8} = \boxed{\frac{15}{19}}$

$\log_2^3 = \frac{1}{\log_3^2} = 1,4 \Rightarrow \log_3^2 = \frac{5}{7}$ -۶

$$\log_{۱۵}^{۶} = \frac{\log_{۳}^{۶}}{\log_{۳}^{۱۵}} = \frac{\log_{۳}^{۲} + \log_{۳}^{۳}}{\log_{۳}^{۵} + \log_{۳}^{۳}} = \frac{\frac{۵}{۸}+۱}{۱٫۵+۱}$$

$$= \frac{\frac{۱۳}{۸}}{\frac{۵}{۲}} = \frac{۱۳}{۲۰} = \boxed{۰٫۶۵}$$

-۷

$$\log_{۸}^{۱۸} = m \quad (۸ \to ۴^{\frac{۳}{۲}}) \Rightarrow \frac{۲}{۳}\log_{۴}^{۱۸} = m \Rightarrow \log_{۴}^{۱۸} = \frac{۳}{۲}m \quad (۱۸ \to ۲\times۳\times۳)$$

$$\log_{۴}^{۲} + ۲\log_{۴}^{۳} = \frac{۳}{۲}m \Rightarrow ۲\log_{۴}^{۳} = \frac{۳m-۱}{۲}$$

$$\left(\log_{۴}^{۲} = \frac{۱}{۲}\right) \Rightarrow \log_{۴}^{۳} = \frac{۳m-۱}{۴}$$

$$\log_{۴}^{۱۲} \ (۱۲ \to ۴\times۳) = \log_{۴}^{۴} + \log_{۴}^{۳} = ۱ + \frac{۳m-۱}{۴} = \boxed{\frac{۳m+۳}{۴}}$$

-۸

$$(۰٫۴)^{۲x-۱} = (۲\times۵^{-۱})^{۲x-۱} = ۲^{۲x-۱}\times۵^{۱-۲x}$$

$$\left(\frac{۱۲۵}{۸}\right)^{x^۲} = (۵^۳\times۲^{-۳})^{x^۲} = ۵^{۳x^۲}\times۲^{-۳x^۲}$$

$$\Rightarrow ۲x-۱ = -۳x^۲ \Rightarrow ۳x^۲+۲x-۱=۰$$

$$\Rightarrow x=-۱ \ , \ x=\frac{۱}{۳} \qquad ۹x+۱>۰ \Rightarrow ۹x>-۱$$

غ ق ق ← $\qquad \Rightarrow x > \frac{-۱}{۹}$

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$$\log_8 (9x+1) = \log_8 4 = \boxed{\frac{2}{3}} \qquad (4 \to 2^2,\ 8 \to 2^3)$$

9-

$$\log_8 b = \frac{1}{3}\log_2 b = \frac{2}{3}(1+a) \Rightarrow$$

$$\log_2 b = \underbrace{2}_{\log_2 4} + \underbrace{2a}_{2\log_2 3} = \log_2 (4\times 3\times 3) = \log_2 36$$

$$\Rightarrow b = 36 \Rightarrow 3b-8 = 100 \qquad \log 100 = \boxed{2}$$

10-

$$-4ax^2 + bx + \frac{1}{4}c = 0 \qquad S = \frac{-b}{a} = \frac{-b}{-4a} = \frac{b}{4a}$$

$$\log_{10} 4 = \frac{4a}{b} \Rightarrow \log_{10} 2 = \frac{2a}{b}$$

$$a = \frac{b+c}{2} \Rightarrow 2a = b+c \Rightarrow c = 2a-b$$

$$\frac{c}{a} = \frac{2a-b}{a} = 2-\frac{b}{a} = 2-2\log_2 10 =$$

$$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = \left(\frac{1}{\sqrt[8]{2}}\right)^{2-2\log_2 10} = \left(2^{-\frac{1}{8}}\right)^{2-\log_2 100}$$

$$= \frac{\left(2^{-\frac{1}{8}}\right)^2}{\left(2^{-\frac{1}{8}}\right)^{\log_2 100}} = \frac{2^{-\frac{1}{4}}}{(100)^{-\frac{1}{8}}} = (0.2)^{-\frac{1}{4}} = 5^{\frac{1}{4}} = \boxed{\sqrt[4]{5}}$$