

سوال ۱

$$\log_n m = a, \log_{mn} m^2 = b, a > 0, [b] = ?$$

$$\frac{\log m}{\log n} = a \rightarrow 1 + \frac{\log m}{\log m + \log n} = b \rightarrow 1 + \frac{a}{a+1} = b \rightarrow [b] = 1$$

سوال ۲

الف) $y = \sqrt{\frac{x}{\log \frac{1}{x}}}$ $\frac{x}{\log \frac{1}{x}} > 0, x \neq 1$ $\frac{1}{-1} + \frac{1}{x} = 0 \Rightarrow D_y = (0, 1)$ $x > 0, x \neq 1$

ب) $y = \frac{\log_2 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$ $x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow \frac{-1}{+1} - \frac{2}{-1} \rightarrow x < -1, x > 2$ $x^2 - 1 > 0 \rightarrow x^2 > 1 \rightarrow x > 1, x < -1$ $\Rightarrow D_y = (-\infty, -1) \cup (2, +\infty)$

سوال ۳

$2 \log_x a + \log_a \sqrt{x} = 2, x=9, a=?$

$$\frac{1}{2} \log_x a = \frac{1}{2 \log_a x} \Rightarrow 2 \log_a 9 + \frac{1}{2 \log_a 9} = 2 \rightarrow \text{تغییری نسبی} \Rightarrow 2 \log_a 9 = 1 \rightarrow a = 3$$

سوال ۴

$\log_2 2 = 1, \log_3 3 = 1, \log_4 4 = 1$ $(\log_2 4)x^2 + (\log_3 4)x - \log_4 5 = 0$ $\Rightarrow 2x^2 + 1x - 1 = 0 \Rightarrow (2x+1)(x-1) = 0 \rightarrow \begin{cases} 2x+1=0 \rightarrow x=-\frac{1}{2} \\ x-1=0 \rightarrow x=1 \end{cases}$ $\Rightarrow \text{اختلاف نسبیها} = \frac{14}{3}$ $\Rightarrow \text{اختلاف نسبیها} = \frac{14}{3} - 1 = \frac{11}{3}$

سوال ۵

$\log_2 8 = 3, \log_3 9 = 2, \log_4 16 = ?$

$$\log_2 8 = 3 \Rightarrow \log_2 2 = \log_3 9 \Rightarrow \log_2 2 = \log_3 3 \Rightarrow \log_2 2 = 1, \log_3 3 = 1$$

$$\log_4 16 = \frac{\log_2 16 + \log_3 16}{\log_2 2 + \log_3 3} = \frac{4 + 2}{1 + 1} = \frac{6}{2} = 3$$

سوال ۶

$\log_3 27 = 3, \log_4 16 = 2, \log_5 125 = ?$

$$\log_3 27 = 3 \Rightarrow \log_3 3 = \log_4 16 \Rightarrow \log_3 3 = \log_4 4 \Rightarrow \log_3 3 = 1, \log_4 4 = 1$$

$$\log_5 125 = \frac{\log_3 125 + \log_4 125}{\log_3 3 + \log_4 4} = \frac{5 + 3}{1 + 1} = \frac{8}{2} = 4$$

سوال ۲

$$\log_8 18 = m, \log_8 12 = ?$$

$$\frac{\log 2 + 2 \log 3}{2 \log 2} = m \rightarrow 2 \log 3 = (2m - 1) \log 2$$

$$\log_8 12 = 1 + \frac{\log 3}{2 \log 2} = 1 + \frac{2m - 1}{2} = \frac{2m + 3}{2}$$

$$\left(\frac{1}{3}\right)^{2n-1} = \left(\frac{125}{8}\right)^{n^2}, \log_8 (9n+1) = ?$$

$$\left(\frac{1}{3}\right)^{2n-1} = \left(\frac{5}{2}\right)^{3n^2} \Rightarrow 1 - 2n = 3n^2 \rightarrow 3n^2 + 2n - 1 = 0 \rightarrow (3n-1)(n+1) = 0 \Rightarrow n = -1 \Rightarrow \text{غیر صحیح}$$

$$\Rightarrow \log_8 \left(9 \times \frac{1}{3} + 1\right) = \log_8 4 = \frac{2}{3}$$

سوال ۹

$$\log_2 3 = a, \log_8 b = \frac{1}{3}(1+a), \log(2b-1) = ?$$

$$\frac{\log 3}{\log 2} = a, \log_8 b = \frac{1}{3}(1+a) \rightarrow \frac{\log b}{3 \log 2} = \frac{1}{3}(1+a) \rightarrow \log b = (1+a) \log 2 \rightarrow$$

$$\log b = \log 2 + \log 2 = \log 4 \rightarrow b = 4$$

$$\log(2b-1) = \log(8-1) = \log 7 = 2$$

سوال ۱۰

$$-f a x^2 + b x + \frac{1}{f} c = 0 \quad \begin{matrix} \log 4 = \text{عکس مجموع ریشه های معادله} \\ a = \text{دستگاه حسب طریقه} \end{matrix} \quad \left(\frac{1}{\sqrt{f}}\right)^{\frac{c}{a}} = ?$$

$$2a = c + b$$

$$\frac{1}{a+b} = \frac{1}{\frac{c}{2a}} = \frac{1}{5} = \frac{1}{\frac{b}{fa}} = \frac{fa}{b} \Rightarrow \frac{fa}{b} = \log 4 \Rightarrow \frac{fa}{2a-c} = \log 4$$

$$\Rightarrow \frac{fa-c}{fa} = \log \frac{1}{4} \rightarrow \frac{1}{f} - \frac{c}{fa} = \log \frac{1}{4} \rightarrow \frac{1}{f} - \log \frac{1}{4} = \frac{c}{fa} \rightarrow \log \frac{1}{f} = \frac{c}{fa}$$

$$\log \left(\frac{1}{f}\right)^{\frac{c}{a}} = \frac{c}{a} \Rightarrow \left(\frac{1}{\sqrt{f}}\right)^{\frac{c}{a}} = \sqrt{\frac{1}{f}} \times \log \frac{1}{f} = \sqrt{\log \frac{1}{f}} = \sqrt{\log 4} = \sqrt{2}$$